Preservice Teachers' Procedural and Conceptual Understanding of Fractions and the Effects of Inquiry-Based Learning on this Understanding

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PRESERVICE TEACHERS’ PROCEDURAL AND CONCEPTUAL UNDERSTANDING OF FRACTIONS AND THE EFFECTS OF INQUIRY-BASED LEARNING ON THIS UNDERSTANDING

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Curriculum and Instruction

by
Hope Marchionda
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Accepted by:
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ABSTRACT

Non-negative rational numbers play a major role in the K-8 curriculum and continue to permeate mathematics content through high school and college in all strands of mathematics. The difficulty that both students and teachers encounter with these concepts is well documented in the literature. This study looked at preservice teacher knowledge and how an alternative means of instruction might improve their conceptual understanding of fractions. To accomplish this task, the study took place in two stages over the course of two semesters. During the first stage, preservice teachers’ conceptual and procedural knowledge of fractions and their associated algorithms were examined through a two-part written assessment and through individual interviews. The results indicate that these participants possess not only weak conceptual knowledge, but weak procedural knowledge as well. Also, when dealing with division, some of the participants’ misunderstandings were due in part to a lack of understanding regarding division of whole numbers. During the second stage of the study, skills and knowledge of preservice teachers who had completed an inquiry-based fraction unit were compared with the skills and knowledge of preservice teachers exposed to a lecture-based unit to determine if one group possesses a better conceptual understanding of fractions and the standard algorithms associated with addition and division. The results indicated that students in an inquiry-based approach to teaching fractions possessed a deeper understanding of fractions and their associated algorithms. Further, the skills of those in the inquiry-based groups were as good as those from the lecture groups, even though
skills were not emphasized during the unit. Another important result was the indication that knowledge retention was greater with the preservice teachers in the inquiry-based section. This study also investigated the impact that inquiry-based lessons have on teacher attitudes as they relate to mathematics and beliefs about mathematics instruction.
ACKNOWLEDGMENTS

This was an adventure that brought on, at times, indescribable emotions and often felt like an insurmountable goal. The feeling of accomplishment, as this project draws to a close, is beyond words. I would not have been able to accomplish my goal of finishing my doctoral degree if it were not for my family and friends who have supported me through the entire process. To all of those who were there for me, I thank you.

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CHAPTER ONE
INTRODUCTION

Non-negative rational numbers play a major role in the K-8 curriculum and continue to permeate mathematics content through high school and college in all strands of mathematics. The difficulty that both students and teachers encounter with these concepts is well documented in the literature. This creates a problem that warrants the attention of those that educate future teachers. Many elementary certification programs require two mathematics content courses for preservice teachers. Within those two courses, students are supposed to become proficient in elementary mathematics so that they can teach the subject. However, these preservice teachers have already completed K-12 education, yet many have difficulty with understanding rational numbers and some still encounter difficulty with procedures involving fractions as well.

This chapter first discusses the current climate as it relates to mathematics education. The next section focuses on teacher quality and how that influences the expectations for preservice teachers’ knowledge as they enter the field of education. The third section addresses student-centered learning and is followed by a brief look at mathematics content and a discussion of rational numbers. The chapter concludes with a statement of the research questions that are the focus of this study.

Current Climate

At the turn of the century mathematics education was influenced by an important document. This document, *Principles and Standards for School Mathematics*, published
by National Council of Teachers of Mathematics (NCTM, 2000) provides schools with guidelines to use when making decisions concerning their mathematics programs. *Principles and Standards* is grounded in the belief that all students should learn meaningful mathematical content and processes with understanding. Since teachers are the main avenue by which children learn formal mathematics in our country, teachers must be effective. To be effective, teachers must possess a deep understanding of the mathematics that they teach (Schoenfeld, 2002; Shulman, 1986). It is not enough to have a superficial understanding of a concept. For example, an elementary school teacher’s (henceforth “teacher”) understanding of a fraction should extend beyond part of a whole. A teacher should be able to think about $\frac{2}{3}$ as a part of a unit and that unit can be an object, a collection of objects, or the distance from 0 to 1 on a number line. In addition, a teacher should also be able to think of $\frac{2}{3}$ as a ratio or as division. However, a teacher’s understanding should not be confined to just conceptual knowledge. Teachers should also be able to apply that knowledge in the context of the classroom and be able to analyze student work and thought processes (Shulman, 1986). The ability to analyze student work and thought processes is important since students often find ways to solve problems that deviate from traditional methods. For example, in a longitudinal study, Carpenter, Franke, Jacobs, Fennema, and Empson (1997) report that “there is mounting evidence that children both in and out of school can construct methods for adding and subtracting multi-digit numbers without explicit instruction” (p. 4). They label the methods students construct as invented strategies. Students may construct these invented strategies in a classroom setting where they are able to share ideas with one another as they try to solve
a problem. During this process the teacher is not one who dispenses knowledge but one who guides toward understanding as students construct their own knowledge (Carpenter, et al, 1997). It is a teacher’s job to be able to analyze these invented strategies to see where children’s misconceptions may be and to also determine the validity of the strategy. The children may have found a way to solve a problem correctly, but his method may not work in all cases. Therefore, a teacher must be able to do more than use an algorithm; she must also understand the concepts well enough to be able to look at mathematics as a dynamic discipline.

Shortly after the release of *Principles and Standards*, legislation was introduced that heightened awareness of the public education system. This legislation, known as the No Child Left Behind Act (NCLB), sets out to accomplish many tasks but specifically seeks to hold schools accountable for “what students know and learn in reading and mathematics.” It also seeks to put highly-qualified teachers in every public school. Both the NCTM’s mission and the NCLB Act place an emphasis on the need for highly trained teachers to be present in all classrooms. The NCLB Act requires states to employ only “highly qualified” teachers by the end of the 2005-2006 school year.

Teacher Quality

In order to employ highly qualified teachers, one must understand what that means. While there is discussion in the literature regarding which factors correlate with teaching effectiveness, there are many contradictions. However, the research does suggest that teacher quality, even though not clearly defined, is the most important educational factor in predicting student achievement (Goldhaber & Anthony, 2003; Darling-Hammond 1998). One critical aspect of teacher quality is content knowledge
(Shulman, 1986). According to a report, *Teacher Preparation Research: Current Knowledge, Gaps, and Recommendations*, that was prepared for the U.S. Department of Education by the Center for the Study of Teaching and Learning at the University of Washington (March 2001), if teachers do not possess a good understanding of the subject matter they will have trouble teaching the material effectively. In a synthesis of research, Wilson, Floden and Ferrini-Mundy (2001) found that elementary and secondary preservice teachers possess knowledge that is rule and procedure dominated but their conceptual knowledge is weak. This can lead to difficulty in explaining why a procedure works. If a preservice teacher is unable to explain why a procedure works then she will probably teacher her students to memorize the procedure which will only perpetuate the problem of placing emphasis on only rules and procedures. While most would not argue that content knowledge is important, there is an indication that at a certain point, further understanding contributes little to teacher effectiveness (NRC, 2001; Wilson, Floden, and Ferrini-Mundy, 2001; Darling-Hammond, 1999). This is due in part to the notion that once a teacher’s expertise has surpassed the demands of the curriculum being taught, the content knowledge advantage decreases. Confounding the difficulties is the research that shows that content knowledge is not the only factor that determines teacher effectiveness. There are indications that pedagogical knowledge, teaching experience, teacher certification, and teacher behaviors can contribute to teaching effectiveness (Darling-Hammond, 1999).

So how much content knowledge must a teacher have to teach mathematics effectively? It is acknowledged by most that many elementary teachers are inadequately prepared to teach mathematics due in part to a lack of conceptual understanding (CBMS,
They draw on their own experiences in K-12 mathematics which have been dominated by rules and procedures (Schoenfeld, 2002). This has led to memorization without understanding (CBMS, 2001; Caine & Caine, 1998). When addressing teacher knowledge, the research often refers to Lee Schulman’s categorization of teacher knowledge. He separates content knowledge into three categories: subject matter knowledge, pedagogical knowledge, and curricular knowledge (Shulman 1986). Requiring teachers to possess each of these three types of knowledge should be a priority since they each play a vital role in the process of teaching. Shulman points out that “the teacher need not only understand that something is so; the teacher must further understand why it is so, …” (p. 9). Skemp recognized two different types of mathematics knowledge - instrumental knowledge and relational knowledge. Instrumental knowledge of mathematics is dominated by rules for performing mathematical tasks, whereas relational knowledge of mathematics is the conceptual understanding of mathematics and the ability to construct various ways to complete mathematical tasks (Skemp, 1978).

A significant gap in many preservice elementary teachers’ mathematics knowledge was created because they learned in a K-12 environment where emphasis was placed on instrumental knowledge but not relational knowledge (CBMS, 2001; Ball, 1989). Take for example dividing fractions. Many preservice teachers can divide fractions because they have memorized an algorithm; however, if asked to explain why this procedure works, many will repeat the process, unable to explain the reasoning behind the algorithm. Shulman called this illusory understanding (Shulman, 2000). As the students progressed through school there was the illusion of learning since they knew
how to perform the algorithm for dividing fractions – invert and multiply. However, the
students could not explain why the algorithm works and therefore did not conceptually
understand the mathematics behind the algorithm. This is just one example of where
mastering the mechanics of the subject does not mean that understanding of the
underlying meanings has taken place (Ball and McDiarmid, 1990). Wilson, Floden, and
Ferrini-Mundy (2001) found in their synthesis of research on teacher education that both
elementary and secondary teachers possess rule-dominated knowledge of basic
mathematics; both groups were weak when asked to explain why an algorithm works.
Being able to explain why something is or why something works is a crucial part of
teaching. In a book prepared by the Conference Board of Mathematical Sciences (CBMS)
entitled The Mathematical Education of Teachers Book (2001) the authors state,
“Prospective teachers need a solid understanding of mathematics; so that they can teach it
as a coherent, reasoned activity and communicate its elegance and power” (p. xi).

According to Ball and McDiarmid, evidence is mounting that all students, not just
preservice teachers, can meet expectations as defined on most high-stakes tests, without
developing a conceptual understanding of the subject matter. This negatively affects a
teacher’s motivation to help students learn in ways that are meaningful. The literature
further reflects that teachers will teach as they themselves were taught (Ball, 1989;
Buchmann, 1989). So if preservice teachers are to enter the classroom and make a
difference, then the cycle of rule-dominated teaching has to change. In order for this to
become a reality in the current climate of reform in K-12 education and the expectation
of having highly trained teachers, one logical place to start is with the education of
teachers.
Many teachers agree that teaching should be more active by using a more hands-on approach; however classroom observations show classrooms where students are not actively involved in the learning process and are instead passive learners (Lowery, 1998). These observations show classrooms where the teacher, not the students, is the center of attention; further, the teacher relies almost completely on a textbook without paying significant attention to her students. Students then spend their time listening to lectures, responding to questions and working from the textbook (Lowery, 1998). In a mathematics class this leads to children following a list of rules to solve a problem or using an algorithm to mimic, without developing a deep understanding of the underlying concepts. Then many of these students who go on to become elementary teachers harbor much math anxiety and believe that they cannot do mathematics because they have tried and failed and have had many poor mathematics experiences.

From talking with students whom I have taught over the years, I have found that many prospective elementary teachers enter into education programs with the dream of becoming great teachers, but worry that they will not be able to teach mathematics, much less teach it effectively. Some even hope that once they can get through the required mathematics courses and obtain their certification, they will be able to get their ideal teaching assignment – one that will require them to teach mathematics only at a level at which they feel comfortable.

When I asked preservice teachers to describe their feelings about teaching mathematics in the future the responses were mixed. One preservice teacher who planned to teach early elementary (kindergarten – 3rd grade) said, “Scared to death! Me not understanding math makes me scared to teach math!!” Another student who was willing
to teach any grade in elementary school said, “I am a bit concerned about my ability to teach math in the future. I am not sure I know the ‘whys.’ I want to convey to my students that math is important but can also be fun and easy sometimes.” A future 3rd or 4th grade teacher said, “When I do teach math, I do not want to be like the teachers I had because even when it seemed nice at the time, in the end it had a negative effect on me. I want to make math fun but also make my students work hard so they don’t end up like me.” These were just a sampling of responses that conveyed an overall negative level of confidence regarding the teaching of mathematics.

The NCTM states that “teachers are key figures in changing the ways in which mathematics is taught and learned in schools” (NCTM, 1991, p. 2). If this is true, should we not make sure that preservice teachers are well prepared to become effective teachers? Part of this preparation is to ensure that preservice teachers are equipped with knowledge they need to help students be more successful. Mathematics education plays an important role in educating future teachers. It can be instrumental in helping teachers understand mathematics concepts and it can shape teachers’ attitudes and expectations about teaching and learning mathematics. If preservice teachers who lack confidence in their abilities to learn and teach mathematics could be part of a mathematics class where they can learn mathematics in a safe environment where questions are welcomed and the process of truly learning the mathematics is not only encouraged but expected, then they might experience a greater understanding of mathematics and in turn develop an appreciation for the discipline. With this newly found appreciation and understanding of mathematics they will be better equipped to break the cycle of teaching rule-bound mathematics.
Student-Centered Learning

In addition to teacher quality, how mathematics is taught is an important issue as evidenced by the current literature. One major theme in the body of literature is that students need to be active in the learning process and that they must construct their own meanings via discovery and experience (Heuwinkel 1996; Jensen, 1998, NCTM, 1991). Having students sit passively in a classroom, though still the norm, is no longer thought to be the best way to learn mathematics (Sousa, 1998; Kruse, 1998; Reardon, 1999, Caine & Caine, 1998; Lowery 1998, NCTM, 1991). Unfortunately, those teachers that believe that an approach where students are active in the learning process is best sometimes have a difficult time implementing this type of instruction (NCTM, 1991). This is attributed to several factors, including their experience as students of mathematics where the strategies employed were geared towards rote learning; consequently, they have had little experience in observing or participating in student-centered learning.

So what constitutes student-centered learning? Many of the student-centered approaches employed in the classroom are grounded in cognitive science that indicates that students must be active in the learning process and that students must construct knowledge based on their prior knowledge. Some of the methodologies taken from this theory are known as active-learning, constructivist learning, or inquiry-based learning. Regardless of what it is called, the common thread is that instead of the teacher telling students what to do and think, the teacher engages the students by questioning, investigating, discussing, and reflecting on the topics of interest.

This study is concerned with inquiry-based learning for two reasons. The first is that preservice teachers need to understand the mathematics they will one day teach. At
this level they have already completed the requirements of K-12 education and started in their post-secondary education; however, as discussed earlier they often lack the conceptual understanding needed to be effective teachers. Now they have enrolled in one of the last math content courses required before they teach mathematics. This presents one of the last formal opportunities for a math educator to help these preservice teachers conceptually understand the elementary mathematics curriculum. If inquiry-based teaching is indeed better at helping students learn, then it should be beneficial to preservice teachers as they try to understand the mathematics they will be teaching. The second reason for a focus on inquiry-based approach is that these preservice teachers will be expected to teach using these or similar methods. Experiencing these methods as learners may help change their attitudes about how mathematics should be taught. Learning mathematics using an inquiry-based approach will allow them to experience first hand how active learning can help their students learn and will provide them with the opportunity to participate and observe how mathematics lessons can be used in their classrooms.

Math Content – Standards (Curriculum and its implementation)

In the *Mathematical Education of Teachers Book (2001)*, the College Board of the Mathematical Sciences (CBMS) recommends that preservice elementary teachers be required to take nine hours of mathematics that are geared towards the fundamental ideas of elementary school mathematics. The CBMS based this decision in part on the idea that “quality of mathematical preparation is more important than quantity” (p. 7). Even with this recommendation many post-secondary institutions require only two elementary education mathematics courses. Regardless of the number of mathematics courses a
A preservice teacher must take, there are certain topics that should be included in the elementary mathematics curriculum for preservice teachers. These topics are addressed in NCTM’s *Principles and Standards for School Mathematics*. The content standards for elementary students are also echoed in preservice mathematics textbooks by many of the leading publishers. In addition, the CBMS is also in agreement about which topics should be covered in elementary mathematics. These topics are divided by the NCTM into five standards: numbers and operations, algebra, geometry, measurement, and data analysis and probability. The process standards, which are also considered essential components by the NCTM and should be utilized in the learning of specific content standards, as listed above, are problem solving, connections, reasoning and proof, representation, and communication. These process standards are essential for learning each of the content standards. For example, problem solving is an excellent tool for learning. Solving problems can lead to developing new understandings about mathematics (NCTM, 2000).

Another document that NCTM published to assist teachers in teaching mathematics is the *Professional Standards for Teaching Mathematics (1991)* (PSTM). These professional standards were written based on the idea that teachers are essential in shifting the way mathematics teaching and learning takes place in the classroom. These standards offer ways in which teachers can transform their classrooms into places where serious mathematical discourse occurs. One way the PSTM assists teachers trying to accomplish this shift is by offering the suggestion that teachers are responsible for “shaping and directing students' activities so that they have opportunities to engage meaningfully in mathematics” (p. 32). This is important because while many of the textbooks used in elementary classrooms are closely aligned with the NCTM’s standards,
90% of the schools in the United States reported following their mathematics textbook very closely (Usiskin & Dossey, 2004). While textbooks are good resources, teachers need to be able to adapt what is done in the classroom based on students’ needs if they are to foster an environment where mathematical reasoning takes place and children are able to solve problems.

Rational Numbers – Fractions

The NCTM’s “Numbers and Operations standard describes deep and fundamental understanding of, and proficiency with, counting, numbers, and arithmetic, as well as an understanding of number systems and their structures” (NCTM, 2000, p.32). As early as kindergarten, students are expected to start developing a basic understanding of common fractions, and they continue to work with fractions each year with an emphasis on operations of rational numbers in upper elementary grades and continuing throughout middle grades. While the focus of the Number and Operation standard, including rational numbers, occurs within the earlier grades, it is important to note that NCTM’s standards are interconnected. Consequently, the importance of learning rational numbers is essential since they permeate other areas of mathematics (NCTM, 2000).

Rational numbers play a significant role in all levels of mathematics but this area often causes difficulty for students and teachers alike (Bezuk & Bieck, 1993; Ball, 1990; Graeber, Tirosh, & Glover, 1989; Simon, 1993; Behr, Lesh, Post, & Silver, 1983; Post, Harrel, Behr, and Lesh, 1991). The CBMS emphasizes that, for teachers to be able to understand the mathematical ideas children develop in an effort to understand rational numbers, preservice teachers must develop a better understanding for themselves (CBMS, 2001). Some teachers also lack procedural knowledge concerning fractions.
Liping Ma (1999) found this to be true: in a study of 21 teachers only 9 were able to correctly use the algorithm for division of fractions and give a complete answer. It has been suggested that before working with the formal algorithms for rational numbers, students should have a deep understanding of rational number concepts and should have developed informal methods, also called invented strategies, to make calculations involving rational numbers (NCTM, 2000; Carpenter, et al., 1997). One way to help students develop a deep understanding of these rational number concepts is by solving realistic problems involving fractions prior to the introduction of formal algorithms (Van de Walle, 2001).

The Research Questions

In the current climate, the push for highly qualified teachers is causing many teacher educators to seek out better methods of teaching preservice teachers. As preservice teachers graduate and move into their careers in education, they need to be prepared to teach the children that will walk into their classrooms. This means that teachers not only need to know the subject matter content but they must also possess the pedagogical knowledge necessary to succeed. The first part of this study informs the larger mathematical community, including curriculum developers, by providing insight into what knowledge preservice teachers possess with regard to fractions. The second part of the study informs teacher educators, specifically mathematics educators as to whether an alternative approach to teaching might support increased conceptual knowledge of fractions. This study also investigates the impact that inquiry-based lessons have on teacher attitudes as they relate to mathematics and beliefs about mathematics instruction.
The following questions directed the work of this study:

1. What knowledge do pre-service teachers possess, prior to taking math content classes, regarding the addition and division of rational numbers? Can they represent the processes symbolically and pictorially and explain the reasoning behind their processes? Can they explain the reasoning behind the standard addition and division algorithm for fractions?

2. Do preservice teachers who have completed an inquiry-based fraction unit possess a better conceptual understanding of fractions and the standard algorithms associated with addition and division than preservice teachers exposed to a lecture-based unit?

3. Does an inquiry-based approach improve preservice teachers’ attitudes about mathematics and does it change their beliefs about how they will one day teach mathematics?

It is necessary to give operational definitions for some key terms.

- **Inquiry-based approach** – Learner centered teaching strategy in which the students’ role is to engage actively in learning, asking questions and investigating ideas. The instructor’s role is to facilitate the learning process. Class time is spent predominately in small groups where students work towards understanding concepts by asking questions, working with concrete learning materials, making connections, and reflecting on processes. Time is also spent discussing their ideas and conclusions with their peers.

- **Lecture-based approach** – Teacher centered teaching strategy in which the majority of class time is spent with the teacher talking and students taking
notes. The students’ role is to listen carefully and to take notes. The students will sometimes ask questions to clarify any misunderstandings and will, at times, answer questions the instructor may have of them.

- **Conceptual Understanding** – Understanding that goes beyond procedural knowledge. This includes being able to represent a mathematical idea pictorially and symbolically, and to explain the concept verbally.
CHAPTER TWO
LITERATURE REVIEW

Introduction

This chapter addresses the relevant literature relating to preservice teachers’ conceptual understanding of rational numbers. In particular, the types of mathematical knowledge a teacher should possess to effectively teach mathematics, mathematics instruction as it relates to teaching preservice teachers and teachers’ understanding of rational numbers are addressed.

Mathematics Knowledge

As mentioned in the preceding chapter, in an effort to improve K-12 public education, the No Child Left Behind Act (NCLB) called for all states to employ only highly qualified teachers. One of the barriers for this goal to become reality is the lack of consensus on what constitutes a highly qualified teacher (Wilson, Floden, & Mundy, 2001). This is an important barrier to overcome since the research does suggest that teacher quality is the most important educational factor in predicting student achievement (Goldhaber & Anthony, 2003; Darling-Hammond 1998).

According to a report, Teacher Preparation Research: Current Knowledge, Gaps, and Recommendations (2001) that was prepared for the U.S. Department of Education by the Center for the Study of Teaching and Learning at the University of Washington, teachers need to possess a good understanding of the subject matter they will teach in order to teach the material effectively. However, there is evidence that teachers’ conceptual understanding is often weak (Wilson, Floden, & Ferrini-Mundy, 2001).
Consequently, many elementary teachers are inadequately prepared to teach mathematics (CBMS, 2001; Wilson, Floden, & Ferrini-Mundy, 2001, NRC, 2001; Ball & McDiarmid, 1990). They draw on their own experiences in K-12 mathematics, which have been dominated by rules and procedures; this encourages memorization without understanding (CBMS, 2001; Caine & Caine, 1998). Even though criticism exists in the literature regarding preservice teachers’ knowledge being dominated by rules and procedures, it is still necessary for them to possess procedural fluency. So if it is necessary, but not sufficient, for teachers to possess procedural knowledge to be effective, then what type of knowledge should a teacher possess to ensure that they teach effectively?

In a report issued by The National Research Council (NRC) in 2001, the term mathematical proficiency is used to summarize what it means for anyone to learn mathematics. Mathematical proficiency includes five interconnected strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. This report, *Adding it up: Helping Children Learn Mathematics*, connects mathematics proficiency to the practice of teaching and it states:

> Just as mathematical proficiency itself involves interwoven strands, teaching for mathematical proficiency requires similarly interrelated components: *conceptual understanding* of the core knowledge of mathematics, students, and instructional practices needed for teaching; *procedural fluency* in carrying out basic instructional routines; *strategic competence* in planning effective instruction and solving problems that arise while teaching; *adaptive reasoning* in justifying and explaining one’s practices and in reflecting on those practices; and a *positive disposition*
toward mathematics, teaching, learning, and the improvement of practice (NRC, 2001, p.10).

These five strands are interconnected and essential to effective teaching. Numerous articles have called attention to subject matter knowledge (CBMS, 2001; Wilson, Floden, Ferrini-Mundy, 2001; Shulman, 1986) and with a push that emphasizes teaching for conceptual understanding (McRel, 2002; CBMS, 2001; Wilson, Floden, Ferrini-Mundy, 2001; NRC, 2001; Ball, 1990; Porter, 1989), also mentioned in the NRC report, it is imperative that preservice teachers are armed with the mathematical knowledge necessary to enter the classroom to do a good job teaching mathematics. This necessitates that teachers have a deep understanding of mathematics so that they can act as facilitators as their students learn mathematics with understanding (Schoenfeld, 2002).

In addition to the NRC reference to mathematical proficiency, there are other references in the literature to the knowledge that teachers should possess. One of the most mentioned is Shulman’s categorization of teacher knowledge. He separates the knowledge teachers must possess into three categories: subject matter knowledge, pedagogical knowledge, and curricular knowledge (Shulman, 1986). He emphasizes that subject matter knowledge is not sufficient knowledge for teaching and that teachers must not only understand the content they will teach but ways of presenting and representing the content to students in a manner that will foster learning.

The literature also references types of mathematical understanding that are tied to the knowledge teachers should possess. One of the most noted is Skemp’s (1978) distinction between mathematical understanding: instrumental and relational. Instrumental understanding involves the rules and procedures of mathematics and is
easier to apply than relational understanding. Relational understanding involves not only knowing how to do something but also why and is more adaptable to a variety of tasks. Skemp believed that a major problem with mathematics education was what constituted mathematical understanding and mathematics knowledge. He believed that this led to mathematics being taught differently and that there was such a difference that they were essentially different subjects, as is evidenced in the following quote. “I used to think that math teachers were all teaching the same subject, some doing it better than others. I now believe that there are two effectively different subjects being taught under the same name ‘mathematics’” (p. 11). While Skemp viewed instrumental (similar to procedural knowledge) and relational understanding (similar to conceptual knowledge) as separate, other researchers believe that both are required for knowing and understanding mathematics. For example, James Hiebert and Thomas Carpenter (1992) stated that looking for which type of knowledge is most important is the wrong approach. They believe that both types of knowledge are crucial and that the question should be how procedural and conceptual knowledge are related.

Even and Tirosh describe Efraim Fischbein’s classification of mathematical knowledge as algorithmic, formal, and intuitive. Algorithmic knowledge includes the rules and procedures for computation and symbolic manipulation and formal knowledge includes axioms, definitions, theorems, and their proofs. The third and final classification of knowledge in this schema is intuitive knowledge and is abstract in comparison to algorithmic and formal knowledge. It is “a kind of cognition that comprises the ideas and beliefs about mathematical entities and the mental models that are used for representing mathematical concepts and operations” (Even & Tirosh, 2002, p. 225). While Fischbein
classified knowledge in three dimensions, he also believed that there was considerable overlap among them.

Regardless of how knowledge is divided and labeled, there are common themes in the literature concerning the knowledge teachers should possess. The first is that teachers should possess a deep conceptual understanding of the content that will allow them to be effective teachers. Another theme is that emphasizing a deep understanding of the content alone is not sufficient. A teacher must also be able to apply her understanding of the content when teaching. It is imperative that teachers understand content knowledge well enough to be able to utilize their knowledge to ascertain when student solutions and explanations are correct, provide explanations, make decisions regarding curriculum, and respond to student questions (McRel, 2002; Floden, McDiarmid, and Wiemers, 1990).

Out of the specific classifications of knowledge and understanding discussed here, Skemp’s relational understanding is the type of understanding that this study is concerned with. Since relational understanding helps students make connections, retention should be increased. In addition, these connections allow for better transfer so that new problems can be solved using known strategies (Hiebert and Carpenter, 1992; Skemp, 1978). While this study looks at how relational understanding might differ between methods, there is optimism that instrumental knowledge will not decline in the process. This study will first establish the strength of students’ instrumental and relational understanding and then determine if an inquiry-based course improves relational understanding more than a lecture-based course.
Another important component of effective teaching is how mathematics is presented in the classroom. Students need to be active in the learning process and must construct their own meanings via discovery and experience (McRel, 2002; Heuwinkel 1996; Jensen, 1998, NCTM, 1991). While many classrooms are still teacher centered where students sit passively in a classroom taking notes, this is no longer thought to be the best way to teach mathematics (Sousa, 1998; Kruse, 1998; Reardon, 1999, Caine & Caine, 1998; Lowery 1998, NCTM, 1991). Unfortunately, there is evidence that even when teachers believe that an approach where students are active in the learning process is best, they have a hard time implementing this type of instruction (NCTM, 1991; Cooney, 1985). Some believe that this can be attributed to the fact that a teacher will teach in the same way she was taught. Others suggest that the method of a teacher utilizes in the classroom is based on her conceptions of mathematics (Skemp, 1978; Thompson, 1984, 1992) or that external demands placed on the teacher such as administrative or curricular dictate how a teacher will present mathematics. This is evidenced in a case study that Thomas Cooney (1985) did of a beginning mathematics teacher. The teacher that was the focus of the case study, Fred, held a problem-solving view of teaching mathematics. However, his classroom practice did not exhibit this view and the case study revealed this conflict. Part of Fred’s problem was that teaching by anything other than the textbook was difficult considering the time demands of a problem-solving approach. This is of importance because Alba Thompson (1984, 1992) found that teachers’ beliefs about mathematics play a role in shaping their teaching behavior. One would then think that if teachers possessed a problem-solving view that they would tend
towards teaching with an inquiry-based approach (Cooney, 1985). As it can be seen with Fred, his problem-solving view of mathematics was not enough. There were other factors influencing his behavior, including student teaching under a teacher whose methods he described as boring. If Fred had a hard time implementing a problem-solving approach to mathematics, would it have helped him if he had been a student in a classroom that was centered on the students and not on the teacher?

In Chapter 1, an initial discussion of student-centered learning started with what student-centered learning is. As noted, many of the student-centered approaches employed in the classroom are grounded in cognitive science that indicates that students must be active in the learning process and that students must construct knowledge based on their prior knowledge. In addition, the common theme among different teaching strategies that are student-centered is that instead of the teacher being the center of attention by lecturing throughout the entire class period, the students become the center of attention by questioning, investigating, discussing, and reflecting on the topics of interest.

Why is the literature teeming with information about changing the practice of teaching mathematics? A weakness of many students is their ability to understand mathematics and use it appropriately (Hiebert, Morris, & Glass, 2003; CBMS, 2001, NRC, 2001). Consequently, there is a call for conceptual understanding of mathematics; it is no longer sufficient for students merely to memorize rules and algorithms. In addition, Hiebert, Morris, & Glass (2003) point out that “the average classroom in the United States reveals the same methods of teaching mathematics today as in the past” (pg. 202).
Research on Instruction

The NCTM has been at the forefront of mathematics reform and believes that learning mathematics should be an active process where students are working towards understanding mathematical ideas and are able to apply and communicate those ideas. The NCTM says that learning should be an active process that stresses the conceptual foundations of mathematics in an effort to make sense of mathematical situations (NCTM, 1991). With this push towards active learning, there have been numerous studies that compare the performance of students in traditional classrooms with the performance of students in reform oriented classrooms. While these studies encompass a wide range of mathematics levels, only studies representing teaching and learning at the elementary and post-secondary levels are reviewed here.

In a year long study Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, and Perlwitz (1991) compared 10 experimental second grade classes with 8 nonexperimental. The instruction in the experimental classes was consistent with constructivist views and was taught using a problem-centered approach. Instruction in the nonexperimental classes was aligned with the Addison-Wesley (1987) second grade textbook. Cobb et al found that the level of computational performance between groups was comparable. However, students in the experimental group possessed higher levels of conceptual understanding. In addition, the experimental group placed more value on the importance of understanding and collaborating with their peers than the nonexperimental group. Furthermore, a byproduct of this research was that at the end of the school year, the teachers that were facilitating the experimental groups held beliefs that were more aligned with constructivist views than the teachers in the nonexperimental groups. The
researchers noted that it is possible to teach using a problem-centered instructional approach and get the desired results and that the research on Cognitively Guided Instruction that is discussed in the following paragraphs adds to the credibility of their results (Cobb, et al., 1991).

This study utilized teachers who volunteered and the teachers in the experimental classes went through training prior to the experiment and had extensive support throughout this study. This is an important consideration, especially when teachers lack experience teaching using a problem-centered approach. These teachers experienced success during the year and in turn volunteered to participate the following year. The teachers’ willingness to participate again is a big indication that this program was a quality program in which they experienced success in teaching using a new approach.

A program that has received much attention in the literature is the Cognitively Guided Instruction (CGI) project directed by Thomas Carpenter, Elizabeth Fennema, and Megan Franke. This project, the CGI Professional Development project, was developed in part because the researchers found that teachers understood children’s thinking to some extent, but that this understanding was not utilized in the decision making process regarding instruction. The professional development episodes operate on two main principles. First there is the utilization of the fundamental ideas underlying the development of children’s thinking about mathematics and second, that the professional development builds on teachers’ existing knowledge (Carpenter, et al, 2000). While the program is not an instructional program in the traditional sense, CGI classrooms possess many similarities (Carpenter, et al, 1999). An example of the way a CGI classroom might
operate is briefly described here and can be found in Learning Number Concepts as Problem Solving (Carpenter et al., 1999).

In a CGI classroom where students are learning computations involving multidigit numbers, word problems provide the basis for instruction. Teachers do not provide ways to solve the problems. Instead, using strategies they have employed to solve similar smaller problems, students model and utilize manipulatives to help reach a solution. Much discussion follows each problem where students share their methods for solving the problems, giving students time to reflect on their own thinking. There is not one correct way to solve a problem. Instead students use a strategy that makes sense to them, eventually moving away from manipulatives and towards abstraction. During this process students learn to add and subtract multidigit numbers without explicit instruction from the teacher. A description of two studies of a CGI classroom is included in the following paragraphs. The importance of these studies is that they show an indication that instruction that is centered on problem solving, builds on students’ prior knowledge, and develops understanding by incorporating the relationship between skills and problem solving produces students who are better at problem solving and just as proficient with computation (Carpenter, et al, 1989).

One study of CGI that was conducted by Albert Villasenor and Henry Kepner and published in 1993 indicates that students in the CGI classrooms significantly outperformed students in non-CGI classrooms in solving word problems, using advanced strategies, and completing number facts. In the CGI classrooms, instruction is centered on problem solving. In the non-CGI classrooms, instruction centered on the textbook where teachers taught specific procedures and students completed problems alone and were
rarely asked to provide explanations into their thinking. This study is of particular interest because it took place in an urban setting and many of the 288 students were minorities. This shows evidence that contradicts the belief that minority or disadvantaged students do not perform well in a conceptually based class (Villasenor and Kepner, 1993).

In another study involving CGI, published in 1989, similar results were found (Carpenter et al., 1989). In this case, analysis of student performance of the students from the CGI class and non-CGI class did not show significant differences in their performance on a computation test. However, the CGI students were able to recall number facts during the number facts interview than the non-CGI students. There were two different problem-solving posttests. The first problem solving posttest contained “simple” addition and subtraction word problems. On this test the significant improvements were seen in those classes who scored the lowest on the pretest. In this group, the CGI students scored higher on the posttest. The second posttest included complex problems with the same operations. In this case the CGI students outperformed the non-CGI students across the board. While this study did not find significant differences in all areas of student performance, this study, combined with the Villasenor and Kepner study, suggests that students do not need to master arithmetic skills before the development of problem solving skills (Villasenor and Kepner, 1993). Instead, problem solving skills can aid in the development of those arithmetic skills. This has important implications for this research study with preservice teachers. Many of them lack not only the conceptual knowledge they need to teach but also the procedural knowledge. Attacking problems to develop similar strategies that their students might one
day invent may give them insight into the process and help them to become knowledgeable in the concepts.

It would be difficult to take one of the studies on CGI classrooms discussed above and generalize the results. However, collectively they support one another and show that there is a strong indication that it is possible to teach children using a problem-centered approach and that students in these classrooms possess more conceptual understanding. In addition, the students show gains in skills as well.

Conceptually Based Instruction (CBI) is another example of how student-centered classes outperform a traditional class. CBI is directed by James Hiebert and Diana Wearne and is similar to CGI in that the focus is on solving problems using a variety of methods including pictures, words, symbols, and manipulatives. Students use their communication skills to share strategies and discuss these strategies to enhance the learning process (Carpenter, et al, 1999). An approach of CBI is to move away from emphasizing the practicing of rules for symbol manipulation to an emphasis on developing conceptual understanding for symbols (Wearne and Hiebert, 1989). Wearne and Hiebert set out to try and help students develop the conceptual understanding so that they could extend their learning to solve a variety of problems instead of memorizing a procedure that can only be applied in specific situations. The study took place in two fourth grade classrooms. The content, decimal numbers, chosen for this research was based on the existing documentation that students have difficulty working with these types of numbers. In addition, there is evidence that students do not have the conceptual knowledge they need and that they memorize procedures and lack the understanding needed for proper use (Resnick et al., 1989; Sackur-Grisvard and Leonard, 1985;
Fischbein, Deri, Nello, and Marion, 1985). In the study lessons were used that were designed to help students develop connections between physical models and symbols, develop procedures to add and subtract decimals, and translate between decimal and fraction form. Interviews and assessments revealed that this type of instruction has some positive outcomes. Incidences where students utilized quantitative reasoning strategies increased throughout the study and these strategies were more likely to lead to a correct answer. While the improvement in transfer was not as pronounced, there was still improvement. In addition, lower achieving students were included in this study and it was observed that these students, as well as the higher achieving students, were able to acquire the processes. However, it was hypothesized that the lower achieving students needed more time to be able to transfer these processes to new tasks (Wearne and Hiebert, 1989).

The study on the CBI classrooms above was small in nature and the instruction was implemented for only a short period of time. Nonetheless, the results were still promising. The study did employ both traditional assessments and interviews to find out what students knew and how they reasoned. This process allowed for more detailed results and the interviews supported the quantitative analysis, adding strength to this study.

While there have been numerous studies on reform oriented instruction in elementary mathematics, the research on teaching mathematics content courses for preservice elementary teachers using reform oriented instruction is not as abundant. However, as was seen with the CGI project, often times the two are intertwined. For example, the CGI project works to help teachers understand student thinking and better
apply that knowledge in the instructional context. This, in conjunction with a student-centered learning environment, in turn affects what students are doing in the class. As a result, the children in the three studies discussed above showed equal or greater gains than their counterparts in the traditional classes. These studies serve as a model for this dissertation in that they demonstrate how altering instruction from teacher-centered to student-centered in an environment of mathematical inquiry can foster the growth of conceptual understanding. This may seem unnatural to compare the education of elementary students to that of preservice teachers, but many of the preservice teachers hold the same misconceptions that K-12 students have about rational numbers (Ma, 1999; Post, Harrel, Behr, & Lesh, 1991; Ball, 1990).

Wilcox, Schram, Lappan, and Lanier completed a study of 23 preservice teachers. The goal of the study was to ascertain how building a community of learners contributes to learning mathematics and learning to teach mathematics. These preservice teachers were enrolled in three nontraditional mathematics courses, a methods course, and a curriculum seminar. These courses were based on course content meeting certain criteria. The content had to engage students by requiring them to actively “do” mathematics by analyzing, abstracting, generalizing, inventing, proving and applying the content. In addition students had to communicate their understanding in multiple ways and participate in a learning community where students and teacher engaged in mathematical inquiry. Over the course of the three mathematics courses, several changes in students’ attitude were noted. One, the students became more confident in their mathematics ability. In addition, students were more willing to engage in mathematical inquiry and apply their knowledge to solve problems that did not fit a specific mold. At the beginning
of the study, many of the students had to adjust to the type of instruction used in the course – a big problem was posed where the solution was not obvious and collaboration was needed to answer questions and discuss the mathematics to work towards a solution. In the end, through observations, interviews, and questionnaires a change was observed in students’ beliefs about the value of group work in learning mathematics. Included in the report about this study was a follow up of two of the teachers, Linda and Allison, as they entered the teaching profession. Linda had experienced more success than Allison at creating an environment where mathematical inquiry could take place. When Linda had trouble leading discussions about problems it was because of a lack of content knowledge. With regard to implementing this type of instruction, she had the support of the administration and some colleagues, but there was some resistance from parents and other colleagues. The main concern was that this type on instruction, by nature, takes more time and therefore, some content had to be eliminated. Allison had also exhibited the desire to create a community of inquiry but had a more difficult time and lacked the support that Linda had from colleagues.

Unfortunately this study does not compare the knowledge gains between this group and a group that was taught in a traditional manner. However, it does offer insight into how building a community of learners centered on mathematical inquiry can contribute to the learning process and in turn affect teachers’ beliefs about teaching and learning. This provides a model for how a content course in mathematics for elementary preservice teachers can enhance students’ mathematical ability. This is evidenced by the students in this study who showed better ability in applying knowledge to unfamiliar problems, increased willingness to engage in mathematical discourse to solve problems,
and better ability at determining the validity of an argument (Wilcox, Schram, Lappan, and Lanier, 1991).

Another study on preservice teacher learning motivates the question regarding teacher beliefs about teaching and learning mathematics and is reported in “The Impact of Enacted Mathematics Curriculum Models on Prospective Elementary Teachers’ Course Perceptions and Beliefs” by Laura Jacobsen Spielman and Gwendolyn M. Lloyd (2004). The research took place in the fall of 2002 in two sections of a mathematics course for elementary students. The control section followed the textbook, *A Problem Solving Approach to Mathematics for Elementary School Teachers* (Billstein, Libeskind, and Lott, 2001) and instruction reflected the philosophy of the textbook. In this section a class typically followed a traditional pattern: homework, lecture, and time to work on problems. In the experimental section, instructional design was centered on the middle school curriculum materials *Mathematics in Context* (MIC; National Center for Research in Mathematical Sciences Education and Freudenthal Institute, 2001) and *Connected Mathematics* (CMP; Lappan, Fey, Fitzgerald, Friel, and Phillips, 1991-1997). Both of these curricula are problem-centered where students engage in problems to help develop understanding of the concepts. The units that were utilized followed the intended design as closely as possible, but diverged when necessary to meet the needs of the preservice teachers’ prior knowledge. In addition, some additional materials were used when a mathematics topic needed to be covered but was not present in MIC or CMP. In this section the class typically started with a homework review which included discussion when questions arose. The instructor redirected the questions to the students but did not provide the answers or explanations. Then instead of listening to a lecture, students spent
time working on the problems or activities. This time was followed by a group
discussion. Students then continued to work on the assignment. To gain insight into
preservice teacher beliefs, a pretest and posttest were given on a Teaching Beliefs
Instrument. There are indications from this study that exposure to reform oriented
curriculum and instruction has an effect on preservice teachers’ beliefs about teaching
and learning.

The researchers report that there were numerous variables to control for so that
the results of the study are hard to generalize. The researchers also had concerns about
the instructor bias and that students may have tried to please the instructor with their
responses. However, even with these limitations there is the indication that the
curriculum and instruction methods chosen for a preservice teachers’ mathematics course
might change students’ beliefs about teaching and learning. This has important
implications for the preservice teachers since many of them will be expected to teach in
environments where students are actively involved in their learning.

Rational Numbers

In a response to the call for reform in the teaching and learning of mathematics,
the first set of standards, *The Curriculum and Evaluation Standards* (1989), were
developed and published by the NCTM. In this document and in the 2000 Standards, the
NCTM outlines standards that can be used as a basis for curriculum development. In the
standards there is considerable emphasis on numbers and operations for the elementary
grades. Students need to develop a good number sense to use throughout their lives and to
provide a foundation for further mathematical study. NCTM believes that teachers should
provide experiences that will help students construct their own number meanings. NCTM
also believes that special emphasis needs to be placed on the concepts of fractions, ratios, decimals, and percents and the multiple representations of these numbers. Students should be able to move among concrete, pictorial, and abstract representations of numbers. “The ability to generate, read, use, and appreciate multiple representations of the same quantity is a critical step in learning to understand and do mathematics” (NCTM, 1989, p. 87). This knowledge of rational numbers also plays an important role in the development of other areas of mathematics such as algebra (NCTM, 1989). This philosophy is echoed when discussing computation. The NCTM continues to support the notion that students need to conceptually understand the mathematics that they are doing. In this process, students need to have a complete understanding of the material and invent strategies that are shared, discussed and validated by their peers and teacher (NCTM, 1989). The emphasis on learning rational numbers is essential since they permeate other areas of mathematics (NCTM, 2000).

As discussed above, rational numbers play a significant role in the development of number and the development of more advanced mathematical topics; however, there is evidence that this area of mathematics causes difficulty for students and teachers alike (Bezuk & Bieck, 1993; Ball, 1990; Graeber, Tirosh, & Glover, 1989; Behr, Lesh, Post, & Silver, 1983; Post, Harrel, Behr, and Lesh, 1991). The CBMS emphasizes that, to be able to understand the mathematical ideas children develop in regard to understanding rational numbers, preservice teachers must develop a better understanding for themselves (CBMS, 2001). This points to the need for preservice teachers to possess both sound procedural knowledge and conceptual knowledge of fractions and their operations.
Liping Ma (1999) found this to be true in a study of 21 teachers. Only 9 were able to correctly use the algorithm for division of fractions and give a complete answer. In her study, teachers were asked to first compute $\frac{3}{4} \div \frac{1}{2}$ and then give a representation for the resulting mathematical sentence. In this study, Ma compared the mathematical understanding of US and Chinese elementary school teachers. While only 43% of the US teachers gave correct or complete answers to the division problem, all of the Chinese teachers were able to give correct, complete answers. Furthermore, when asked if the algorithm made sense, the Chinese teachers, but not the US teachers, were able to elaborate. During this process, the US teachers all referred to the invert and multiply algorithm whereas the Chinese teachers proposed additional approaches. Shockingly, with regard to representing division of fractions, 16 had misconceptions in their story problems, 6 could not create a story, and only one of the US teachers presented a conceptually correct representation. However, this correct representation posed another problem, which the teacher realized. The representation, using the context of children and Twinkies, gave a result of $3\frac{1}{2}$ children. This is problematic because it is not a real life number since one would never have a fraction of a person. To further exemplify the lack of understanding some of the US teachers had, there were opportunities for these teachers to realize that the conceptualizations were incorrect. Of the 16 that made up a story to conceptualize $\frac{3}{4} \div \frac{1}{2}$, 9 correctly computed the answer. Of these 5 teachers noticed the discrepancy between the computation and the answer to their incorrect
story problem but were still unable to come up with a correct representation of the division problem. This study has important implications for the types of knowledge that preservice teachers should possess to enter the teaching profession. Overall the US teachers were so lacking in conceptual knowledge that they were unable to create representations for division of fractions. As Ma points, “Even their pedagogical knowledge could not make up for their ignorance of the concept. Circular foods are considered appropriate for representing fraction concepts. However, as we have just seen, the representations teachers generated with pizza or pies displayed misconceptions” (Ma, 1999, p. 70).

Ma’s study revealed that US teachers viewed mathematics as an “arbitrary collection of facts and rules in which doing mathematics means following a set of procedures set-by-step to arrive at answers” (pg. 123). This was in contrast to the Chinese teachers who were not only interested in using an algorithm but also how it works. The results show that US teachers possessed deficits in conceptual knowledge, especially in comparison to Chinese teachers. This study was small in scale in terms of the number of teachers interviewed. In addition, it would be beneficial to have more specific information about all of the teachers that participated in the study. Without a better cross section of teachers representing both countries, it is not clear if these results are representative of all teachers. However, other studies point to a similar conclusion.

Another study that focused on the mathematical understanding of teachers (Post, Harel, Behr, and Lesh, 1991) shows similar evidence of teachers’ lack of mathematical knowledge. There were 218 participants in this study and all were 4th, 5th, and 6th grade teachers from Minnesota and Illinois. A three part assessment was given to the teachers.
Part 1 consisted of short answer problems, part 2 required teachers to give pedagogical explanations of the solutions they generated, and part 3 was an interview about rational number concepts. What the researchers found was that between 10 and 25 percent of the teachers incorrectly answered problems that were considered to be of an elementary level. Even more troubling is that as many as half of the teachers answered some of the questions incorrectly. For example, between 40 and 50 percent were unable to order a list of fractions from smallest to largest. This problem was a 1979 item pulled from the National Assessment of Education Progress (NAEP) student assessment. On another problem, in dealing with fraction equivalence, approximately 65 percent of the teachers were unable to correctly answer $\frac{8}{15} = \frac{4}{5}$.

This study was comprehensive in that two thirds of the schools in a particular district were involved. In addition, since the nature of the study was to determine factors associated with learning rational number concepts, only teachers who were teaching mathematics at the time were included in the study. The teachers were required to participate which could confound the results. By requiring teachers to participate, there was a better representation of teachers. A volunteer study would tend to have teachers with a vested interest in education so the data might be skewed in a positive direction. However, there could also be a negative effect since some less interested teachers might harbor negative feelings toward the study and not participate fully. To get an idea about how these results might compare to other districts across the nation, it would have been beneficial to have more information regarding the demographics of the school district and the teachers that were part of the study.
In a 1990 article Deborah Ball reported on 19 preservice teachers’ understanding of division. This report was part of a larger study that looked at preservice teachers’ mathematical and pedagogical knowledge. The group of participants included 10 elementary and 9 secondary preservice teachers. While the study looked at other aspects of division, only the analysis that dealt with division of fractions is included here. In her interviews with the participants she gave them the division problem $\frac{3}{4} \div \frac{1}{2}$ and asked them how they would solve the problem. Some of the participants tried to come up with application problems. This process revealed that the participants “framed the problem in terms of fractions, but also that many were uncomfortable with fractions as quantities” (Ball, 1990, pg. 134). Seventeen participants were able to reach the correct answer, but only 5 were able to generate a correct representation for the division problem. The 5 participants who gave correct representations were all secondary preservice teachers. One common error was that participants viewed this problem as division by 2 and not by $\frac{1}{2}$.

One of Ball’s conclusions was that many of the participants encountered difficulty because their view of division was limited to “forming a certain number of equal parts” (Ball, 1990, pg. 140).

These studies involving rational numbers demonstrate that there is evidence that teachers lack conceptual knowledge of rational numbers and their operations. This certainly provides a problem within the classroom. If rational numbers are a cornerstone of mathematical thought, then teachers should be prepared to teach them. As evidenced in the literature, teaching should extend beyond requiring students to memorize definitions or algorithms without understanding. This means that teachers must possess the necessary
understanding to make decisions regarding curriculum and instruction in an effort to help students understand. This lack of conceptual understanding by mathematics teachers was one of the motivating factors behind this research. The other was to ascertain if inquiry-based instruction might help preservice teachers develop a better conceptual understanding of fractions. In the following chapter, the methodology for this research study is discussed.
CHAPTER THREE
RESEARCH METHOD

Introduction

In this chapter I provide information about the research method that was employed in answering the following questions:

1. What knowledge do pre-service teachers possess, prior to taking math content classes, regarding the addition and division of rational numbers? Can they represent the process symbolically and pictorially and explain the reasoning behind their processes? Can they explain the reasoning behind the standard addition and division algorithm for fractions?

2. Do preservice teachers who have completed an inquiry-based fraction unit possess a better conceptual understanding of fractions and the standard algorithms associated with addition and division than preservice teachers exposed to a lecture-based unit?

3. Does an inquiry-based approach improve preservice teachers’ attitudes about mathematics and does it change their beliefs about how they will one day teach mathematics?

This chapter is divided into four main sections. It begins with a description of the participants and the setting and how the experimental sections were chosen for the study. The second section is a description of the inquiry-based curriculum implemented in the experimental section of the study. The next section is a description of the method used for
data collection. The last section discusses the procedures used for organizing and analyzing the data that were collected over the two semesters of the study.

Participants and Setting

The Setting

This study was conducted at a major university in the Southeastern United States and will be referred to as Southeastern University. While the Department of Curriculum and Instruction assumes the primary responsibility in educating future teachers, that responsibility is shared by the entire university as discussed below. The major with the greatest number of students enrolled in the university is elementary education; therefore, much of the university has a hand in preparing future elementary schools teachers. Here at Southeastern University, the Mathematics Department assumes the responsibility for teaching the mathematics content for preservice elementary teachers (K-8) and preservice secondary teachers (9-12).

Southeastern University requires preservice elementary teachers to take two mathematics content courses specifically designed for elementary teachers. Prior to enrolling in this two course sequence, students must have a “C” or better in a general education mathematics course at the collegiate level, which is College Algebra or higher. Once students have completed this prerequisite, they may enroll in the first course of the sequence. In order for preservice teachers to be able to proceed to the second course in the sequence, they must pass the first course with a “C” or better. In this two course sequence, the mathematics department adopted a more stringent grading scale in an effort to ensure that students are better equipped mathematically. This means in order to earn a “C” a student needs at least 76% in the class instead of 70%.
These courses, henceforth referred to as Math I and Math II, cover a wide scope of K-8 mathematics. This, by nature, lends itself to a lecture style format where an instructor can address many objectives in a relatively short time frame. This agenda is ambitious in its attempt to help preservice teachers understand the mathematics they will teach. These two courses parallel the chosen text, *Mathematics for Elementary Teachers: A Contemporary Approach* by Gary Musser, William Burger, and Blake Peterson, and cover many of the NCTM’s standards for K-8 mathematics. Math I is primarily concerned with the NCTM’s Numbers and Operations content standard and Math II covers measurement, geometry, and data analysis and probability. Although lecture has been the dominant instructional strategy for Math I, some of the instructors that teach the course on a regular basis introduce some manipulatives during the semester. However, with the amount of material that is covered during this semester long course, there is typically not a great deal of time spent using the manipulatives or using an inquiry or constructivist approach.

*The Participants*

The education department suggests that students take Math I and Math II during their sophomore year. However students do not always follow this recommendation and many students take the course as freshmen or juniors. Most students enrolled in Math I range in age from 18 to 21, though a few are older, non-traditional students. Few of the students have taken many education classes since they must complete their general education classes and content classes before being allowed to enter the education block. The block is divided into two semesters and must be completed prior to student teaching. Typically, students enroll in the block during the second semester of their junior year and
first semester of their senior year. During these two semesters students also take the majority of the required education courses. These courses include foundation courses and methods courses for the main disciplines taught in elementary school (reading, social studies, science and mathematics), and a senior project. These courses are typically taken after Math I and Math II. Preservice teachers spend the last semester student teaching.

Inquiry-Based Curriculum

The content in the lecture-based sections (control group) of this study were closely aligned with the textbook *Mathematics for Elementary Teachers: A Contemporary Approach* by Gary Musser, William Burger, and Blake Peterson. The lecture-based section was taught using traditional methods where the instructor was the center of attention and the majority of class time was spent with the teacher talking and students taking notes. Student participation was centered on asking questions to clarify any misunderstandings and answering questions the instructor asked. In the traditional section, the objective was to make it through nine chapters of the textbook in the semester. At Southeastern University, most instructors find this difficult to manage in one semester.

The inquiry-based sections (experimental group) were also conducted using traditional methods until the unit on fractions. At that time, instruction changed from lecture-based to inquiry-based. In this research study, inquiry-based learning referred to a classroom that was learner-centered where the students’ role was to engage actively in their own learning where they develop their own understandings though investigation while the instructor’s role was to facilitate the learning process. Class time was spent predominately in small groups where students worked towards understanding concepts by
asking questions, working with concrete learning materials, making connections, and reflecting on processes. The groups in each experimental section were student selected, but consistent throughout the unit. Time was also spent discussing their ideas and conclusions with their peers during small group and whole class discussions. The lessons were intended for use in a class that lasts at least 80 minutes.

By nature inquiry takes more time than lecture: therefore, content had to be prioritized and some of the peripheral content had to be eliminated in both sections. This was done in conjunction with the mathematics education faculty in the department. Once the big ideas were chosen for the experimental group for the fraction unit, I adapted and developed inquiry-based activities for the classroom and for homework. The lessons were then piloted in the fall of 2005. Both instructors took part in the pilot and therefore, the pilot served as professional development for the instructor that assisted in this study. Since she did not have experience teaching in an inquiry-based learning environment, both the observations she made during this semester and her participation in teaching the lessons were invaluable. During and after the pilot, the other instructor and I worked to improve on the lessons based on our experiences during the pilot, observations of students, and informal student feedback. During the spring semester when data were collected for this research, I observed every inquiry-based lesson the other instructor taught and a fidelity checklist was used to help ensure that she was doing what was expected in the inquiry-based sections. During the pilot and the implementation of the lessons, the other instructor and I held daily meetings about what happened in the previous lesson and what needed to happen in the next lesson. These conversations also
addressed what to expect in the next lesson and how discussions should be facilitated during each lesson.

The lessons were broken down into four main categories: general fraction knowledge, addition/subtraction of fractions, multiplication of fractions, and division of fractions. The basis for the lessons grew from departmental expectations concerning the most important concepts for preservice teachers to learn. With this as a basis, I pulled from my experience teaching fractions in this course as well as other courses I had taught at the secondary and post-secondary levels. In addition, I used ideas from well-respected professionals in the field of teaching mathematics, such as Susan Lamon and John Van de Walle. Once the lessons were written, I elicited and received feedback from two mathematics educators.

The lessons that addressed general fraction knowledge focused on the concept of a fraction, the importance of the unit, equivalent fractions, relative amounts, and ordering fractions. The main focus of the addition/subtraction lesson was to answer the question of why we need a common denominator when adding fractions with a standard algorithm. To help students answer this question and understand this process, they were given problems to solve using pictures. The focus on the multiplication lessons was to understand different interpretations of fraction multiplication that involve fractions and to use models to understand the standard algorithm for multiplying fractions. In addition, students connected multiplication of mixed numbers, the rectangular array approach, and the F.O.I.L method for multiplication. The last lesson focused on why we invert and multiply when we divide fractions. The focus was to get preservice teachers to
understand the algorithm conceptually. Two of the lessons are included in Appendices and are described below.

Appendix A includes a lesson that enhances the conceptual understanding of fractions and continues to build on the idea of the unit. The first part of the lesson focuses on relative amount and how this concept can complicate the understanding of fractions. The lesson continues exploration of the unit but does so through the investigation of fraction relationships with a length model. Equivalent fractions are also explored by using manipulatives while exploring a length model. The last task in this lesson guided students to think about fractions in relation to one another in an effort to order them without relying on an algorithm.

Appendix B includes a lesson on adding and subtracting fractions that uses contextual problems to develop an understanding of addition and subtraction of fractions using pictorial representations. Then these pictorial representations are used to build towards an understanding of why it is essential to have common denominators when using the standard algorithm for addition and subtraction of fractions.

Data Collection

Data collection took place during two semesters. The data collection during the fall semester of 2005 occurred in four sections of Math I at the beginning of the semester and was used to answer question 1 of this study. Data collection that occurred in the spring of 2006 took place in four sections of Math I with two different instructors, and it was used to answer questions 2 and 3. During the spring semester, each instructor taught one section using a traditional lecture-based approach (control) and one section using an inquiry-based approach (experimental); each section of the course was limited to 32
students. For each instructor, these courses met on the same days of the week. Instructor A taught her sections of Math I on Monday/Wednesday/Friday (MWF) and Instructor B taught her sections on Tuesday/Thursday (TR). The MWF sections met for 55 minutes each class period and the TR sections met for 80 minutes each class period. Each instructor taught the control group first and the experimental group second. Although the students chose which section to sign up for, they were not aware of the research study so they could not choose to be part of the control or experimental section.

Data collection took place beginning in August 2005 and ending in May 2006. Data were collected through observations, surveys, interviews, and student work. Since these data were collected during two different semesters, each semester is addressed separately in this section.

Fall 2005

Assessment

In an effort to determine what knowledge preservice teachers possess regarding fractions and their operations, all participants were given a two-part assessment on the first day of class. Part 1 of this assessment included items obtained from the Trends in International Mathematics and Science Study (TIMSS). These items were released from the 1999 TIMSS assessment and were chosen from the content domains that are relevant to Math I content. These questions were a mixture of multiple choice and free response questions. Part 2 of this assessment included two problems that were used to gain insight into the preservice teachers’ conceptual knowledge of fractions and their operations. The addition problem and one of the division problems was adapted from *Elementary and Middle School Mathematics* (2001) by John Van de Walle. I wrote the second division
problem. With these problems, students were asked to solve them two different ways: using pictures and using algorithms.

**Interviews**

In addition to the assessment at the beginning of the semester, interviews were conducted from a list of volunteers that was obtained on the first day of class. In an effort to gather information from a diverse group of students, I ranked these volunteers based on their past performance. I utilized all available information which included ACT/SAT scores, Math Placement Exam scores, and grades from their previous post-secondary mathematics courses. Working with each course section separately, I divided the students into thirds based upon these measures. I then randomly chose three students from each group in each section. Since it is not unusual to have students repeat the course, I chose one student from each group who had already taken the course. Participants were contacted from this list of volunteers to coordinate an interview time that would be convenient for the participant. The interview focused on establishing a rapport and identifying what each of the preservice teachers believed about mathematics and teaching mathematics. This interview also focused on the preservice teachers’ perceptions of their ability to do mathematics, specifically fractions, and their ability to teach mathematics. The interview also sought to determine what these participants knew about fractions and the addition and division algorithms. Interviews were tape recorded and transcribed.
Spring 2006

*Student Work/Assessments*

In an effort to establish which section, control or experimental, possessed better conceptual understanding of fractions at the end of the semester, a pretest and posttest were administered. These assessments were the same as the assessment given in the fall of 2005 and included items obtained from the Trends in International Mathematics and Science Study (TIMSS) and two word problems as described in the previous section. During the semester, student work related to fractions was collected in the form of journals, homework, quizzes, and a chapter test. At the end of the semester the fraction content on the final exam served as the cumulative assessment for content knowledge.

*Observations*

Observation data were collected via videotapes. In the experimental section, the class was divided into groups of two to four that were chosen randomly. I videotaped two groups during the unit on fractions. One group was videotaped on a regular basis to maintain consistency and to look for trends. A second group, which changed with each videotaping, was videotaped to provide a comparison. In the control section, students occasionally worked in groups but since the course was predominantly lecture, videotaping was typically of the entire class. Both sections were videotaped during the entire unit on fractions, which consisted of seven class meetings. Following each lesson the instructors reflected on the lesson; these reflections were used in conjunction with the videotape. In addition, the groups that were the focus of the videotaping were asked to reflect on the lesson as part of a journal entry. The focus of the observations was to document the exchanges between participants or between instructor and participants that
supported the learning process. This information was used to corroborate the findings from other data collected.

**Surveys and Interviews**

At the beginning of the semester a survey was given to determine students’ beliefs and attitudes about mathematics and about teaching mathematics. The first part of this survey was a modified Fennema-Sherman Mathematics Attitude Scale (FSMAS) and it was utilized to obtain information about students’ attitudes about mathematics. This version of the FSMAS was modified by Diana Doepken, Ellen Lawsky, and Linda Padwa. The major modification was to the length of the original attitude scale that was developed in 1976 by Elizabeth Fennema and Julia Sherman (Fennema & Sherman, 1976). The complete FSMAS included nine different scales each with 12 items. These scales could be used individually or grouped in any combinations (Fennema & Sherman, 1976). The modified version was shortened to 47 questions from four of the original 9 scales. This was an important factor in choosing this modified version. The original 108 question scale is long and students could lose interest in responding and invalidate the results. These scales used in this modified FSMAS are addressed in Chapter 4. The second part of the survey was adapted from a survey developed and used by Donna Diaz (2004) to examine teachers’ knowledge and attitudes for teaching. She developed this survey based on part of her dissertation. It was written as pre/post retro survey. For the purpose of this study, I was interested in the opinion of the participants only at the time they completed the survey. Therefore, the survey was not administered as a pre/post retro but as a traditional survey instead. This two-part survey was then given again at the end of the semester to determine if there had been a change in students’ beliefs and attitudes.
The initial part of this survey was designed to gain insight about previous experiences relating to learning mathematics. The second part related to how to teach mathematics. The surveys were given in both sections of the study to compare the changes with the experimental group to those of the control group.

Formal interviews were conducted at times that were convenient to the participants. During this semester, there were not enough volunteers to rank, based on prior performance, within each section. As a result, volunteers were chosen at random within each section until at least three interviewees from the list of volunteers agreed to participate. There were a total of 13 interviewees for the spring semester.

There were two interviews during this semester. The first interview initially focused on establishing a rapport. Both interviews then focused on identifying what each of the preservice teachers believed about mathematics and teaching mathematics. There was also a focus on their perceptions of their own ability to do mathematics, specifically fractions, and their ability to teach mathematics. The second part of each interview focused on students’ conceptual knowledge of fractions to ascertain what they knew about fractions and their operations. All interviews were tape recorded and transcribed.
**Stages of Data Collection**

The following table provides an outline of the data collection that took place over the course of this study.

Table 3.1: Stages of Data Collection

<table>
<thead>
<tr>
<th>Data collection stage</th>
<th>Dates</th>
<th>Types of data</th>
<th>Purpose of collection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage I</td>
<td>August-September 2005</td>
<td>Pretests and interviews</td>
<td>To determine what preservice teachers knew before completing their math content courses.</td>
</tr>
<tr>
<td>Stage II</td>
<td>January 2006</td>
<td>Pretest, initial Interview, and Surveys</td>
<td>To establish base-line data concerning preservice teachers’ procedural and conceptual knowledge and their beliefs about teaching and learning mathematics</td>
</tr>
<tr>
<td></td>
<td>March - May 2006</td>
<td>Unit Test, Final Exam, Posttest, Surveys</td>
<td>Data collected regarding content knowledge was utilized in two ways. One was to compare the control and experimental groups to look for difference in improvements to content knowledge. The second was to see if there was a difference in what students knew at the beginning of stage II versus the end of stage II. Data collected regarding beliefs was used to see if a change occurred over the course of semester and if the changes were the same in each group.</td>
</tr>
<tr>
<td>Ongoing</td>
<td>January – May 2006</td>
<td>Journal entries, observations, and student work</td>
<td>Data collected utilizing these avenues were used to corroborate data collected during each of the stages.</td>
</tr>
</tbody>
</table>
Data Analysis

In order to address all questions, I utilized both qualitative and quantitative methods to analyze the data.

Organizing the Observation Data

Since one purpose of the study was to ascertain whether an inquiry-based approach leads to a better understanding of the concepts, the observations were helpful in documenting interactions that supported the learning process. The instructors’ post lesson notes helped determine what the instructor was thinking during these interactions. These notes, student reflections, and transcribed portions of the videotape were coded based on a method outlined in Analyzing & Interpreting Ethnographic Data by Margaret LeCompte and Jean Schensul (1999). In order to interpret the data, I conducted an item-level analysis. To accomplish this, I read each set of instructor notes, reflections and transcribed video to identify pertinent information. As similarities and themes emerged, I created codes to assist in marking items that related to learning mathematics so that they could be used to corroborate evidence of learning and conceptual understanding.

Organizing the Interview and Survey Data

Surveys were administered at the beginning and end of the study. Part 1 of the survey, the modified Fennema-Sherman, was analyzed with a key that was developed for use with the survey. These data were then analyzed in SAS using the General Linear Model (GLM) procedure and repeated statement (Cody & Smith, 1997). Part 2 of the survey was scored in a similar fashion to the Fennema-Sherman portion, but each question was scored on a scale of 1 to 3. The results were then analyzed using the same
procedure in SAS. Additional details regarding the scoring process for both portions of this survey are provided in Chapter 4.

Interviews were analyzed two different ways. For the section relating to beliefs and attitudes, any response related to a participant’s beliefs about teaching or learning mathematics was rated from 1 to 5. One corresponded to a negative view, 3 to a neutral view, and 5 to a positive view. The portions of the interviews relating to math content were coded based on the level of proficiency a student possessed. This coding scheme was adapted from a study completed by Bryan (1999) and described in “The Conceptual Knowledge of Preservice Secondary Mathematics Teachers: How Well Do They Know the Subject Matter They Will Teach?” Responses to interview questions regarding mathematics content were examined for procedural knowledge as well as conceptual knowledge. Table 3.2 outlines and gives a description of the codes that were used in conjunction with concepts regarding fractions.
Table 3.2: Interview Content Codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRO-0</td>
<td>Showed no procedural knowledge</td>
</tr>
<tr>
<td>PRO-</td>
<td>Showed flawed or incomplete procedural knowledge</td>
</tr>
<tr>
<td>PRO+</td>
<td>Showed solid procedural knowledge</td>
</tr>
<tr>
<td>PIC-0</td>
<td>Offered no pictorial representation</td>
</tr>
<tr>
<td>PIC-</td>
<td>Offered flawed pictorial representation</td>
</tr>
<tr>
<td>PIC+</td>
<td>Offered sound pictorial representation</td>
</tr>
<tr>
<td>VER-0</td>
<td>Offered no verbal explanation</td>
</tr>
<tr>
<td>VER-</td>
<td>Offered flawed verbal explanation</td>
</tr>
<tr>
<td>VER+</td>
<td>Offered sound verbal explanation</td>
</tr>
</tbody>
</table>

When organizing the results of the interviews, I adapted these codes for readability in table format. These adaptations are described in Chapter 4 when the results are reported.

**Organizing Student Work**

The pretest, quiz, unit test, posttest, and final exam were all scored using their corresponding rubrics. On the unit test and the final exam, only the problems that tested conceptual knowledge of fractions were scored for this study. All of these assessments were photocopied during the semester before they were graded. Then at the end of the semester each of the assessments was graded double blindly to protect against researcher bias. Quantitative measures were then utilized to answer the specific questions relating to performance that this study set out to answer. Student journals and interviews were used to corroborate the quantitative results.

The quiz, unit test, final exam, and difference scores (post minus pre) for both parts of the pretest and posttest were analyzed using a multivariate analysis of covariance.
(MANCOVA). For the MANCOVA, the independent variables were the method (control or experimental) and instructor (A or B) and the dependent variables included each of the assessments. ACT scores were used as a covariate. In an effort to increase the possibility of detecting a difference in the groups, I selected an alpha level of 0.10.

I hypothesized at the beginning of this study that there would not be a significant difference in overall assessment scores when comparing the experimental and control sections. To accommodate this hypothesis, I planned to conduct additional analysis on those students who scored less than 80% on Part 1 of the pretest. This consisted of the TIMSS questions which contained numerous problems that required only procedural knowledge. However, there were no ceiling effects in the full sample, so no additional analysis was needed on the reduced sample.

Summary

The following gives a synopsis of what data I used to answer each of my research questions.

1. What knowledge do pre-service teachers possess, prior to taking math content classes, regarding the addition and division of rational numbers? Can they represent the process symbolically and pictorially and explain the reasoning behind their processes? Can they explain the reasoning behind the standard addition and division algorithm for fractions?

I utilized information gathered during the first stage of this study. The data came from the pretest given at the beginning of the semester as well as interviews conducted during the first two weeks of class. The analysis of these data was quantitative and qualitative.
2. Do preservice teachers who have completed an inquiry-based course possess better conceptual understanding of fractions and the standard algorithms associated with addition and division than preservice teachers from a lecture-based course?

Information gathered during the second stage of the study was utilized. This included a pretest, a posttest, a quiz, a unit test, the final exam, and interviews. The pretest, posttest, and other test measures were analyzed using a MANCOVA. Other forms of student work and the interviews were used to corroborate the results of the MANCOVA. Item level analysis was also used with interview data to investigate preservice teachers’ procedural and conceptual knowledge.

3. Does an inquiry-based approach improve preservice teachers’ attitudes about mathematics and does it change their beliefs about how they will one day teach mathematics?

Data gathered during the second stage of the study were used to answer this question. The surveys from the beginning and end of the semester, interviews, and journal entries were used here. The surveys were analyzed using repeated measures and qualitative analysis was used with the interviews and journal entries.

The results for this data analysis are provided in Chapter 4.
CHAPTER FOUR
RESULTS

Introduction

In this chapter I examine the data collected from the preservice teachers at Southeastern University. The chapter is divided into three main sections that are aligned with the questions I set out to answer with this study. Data from the fall of 2005 are used to substantiate what prior knowledge the pre-service teachers possessed and data gathered during the spring of 2006 are used in the next two sections to substantiate the findings concerning preservice teacher learning and any changes in their beliefs regarding teaching and learning throughout the semester.

Question One: Prior Knowledge

In this section, I provided the analysis of the assessment given at the beginning of the semester as well as interview data to answer the following questions: What knowledge do pre-service teachers possess, prior to taking math content classes, regarding the addition and division of rational numbers? Can they represent the processes symbolically as well as pictorially and explain the reasoning behind their processes? Can they explain the reasoning behind the standard addition and division algorithm for fractions?

Participants

The first stage of data collection occurred in the fall semester of 2005 and focused on what pre-service teachers knew about fractions and their operations prior to successfully completing their math content courses. Ninety-four students took part in this
phase of the data collection during the fall semester of 2005. A summary of the participants’ major of study is shown in Table 4.1. As shown in the table, the majority of the participants were elementary education majors. Math I is a requirement for all the participants except for the one Early Elementary Education Major. Out of the 94 participants, 13 had previously completed at least one semester of this course with a grade of D or F.

Table 4.1: Participant’s Major of Study

<table>
<thead>
<tr>
<th>Major</th>
<th>Number-declared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary Education</td>
<td>72</td>
</tr>
<tr>
<td>Middle Grades Education</td>
<td>7</td>
</tr>
<tr>
<td>Exceptional Education</td>
<td>13</td>
</tr>
<tr>
<td>Early Elementary Education</td>
<td>1</td>
</tr>
<tr>
<td>General Studies with an Emphasis in Education</td>
<td>1</td>
</tr>
</tbody>
</table>

Assessment

At the beginning of the semester each of the participants took a two-part assessment. The first part consisted of 23 questions that came from the TIMSS website and the second part consisted of two application problems which the students were asked to solve in two different ways. The TIMSS questions were released from the 1999 test that was given to 8th grade students. Consequently, there is a possibility that some of the students in this study were in 8th grade when these questions were administered in 1999.
As I look at the results for the questions that were given in this study, I will at times compare the results here to the results in 1999 for 9000 8th graders worldwide.

Assessment, Part I

All of the questions on Part I of the assessment were chosen from the content domain of fraction and number sense. Each one of these questions falls into one of the cognitive domains that are listed in Table 4.2. This classification system allowed for a natural way to look for patterns where the participants had the most difficulty on the assessment and where there were gaps in their prior knowledge. Table 4.3 includes a topic description of all 23 questions with a breakdown of the cognitive domain, question content, number of participants answering correctly, and the percentage of 8th graders that answered correctly in the United States and Internationally. To get a better sense of what these pre-service teachers knew coming into the math content courses, I will take a closer look at the types of questions that were asked and types of knowledge they required. To do this, Table 4.3 has been broken down into four separate tables so that it will be easier to see what is going on with each cognitive domain.

Table 4.2: Cognitive Domains for Questions on Part I of Pretest

<table>
<thead>
<tr>
<th>Cognitive Domain</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investigating and Solving Problems</td>
<td>I&amp;SP</td>
</tr>
<tr>
<td>Knowing</td>
<td>K</td>
</tr>
<tr>
<td>Using Complex Procedures</td>
<td>UCP</td>
</tr>
<tr>
<td>Using Routine Procedures</td>
<td>URP</td>
</tr>
</tbody>
</table>
Table 4.3: Scores on Part I of Pretest

<table>
<thead>
<tr>
<th>Cognitive Domain</th>
<th>Question</th>
<th>Number of Correct Responses out of 94</th>
<th>Percentage Answering Correctly</th>
<th>8th graders US</th>
<th>8th graders International</th>
</tr>
</thead>
<tbody>
<tr>
<td>I&amp;SP</td>
<td>Proportion</td>
<td>68</td>
<td>72.34</td>
<td>68</td>
<td>69</td>
</tr>
<tr>
<td>UCP</td>
<td>Equivalence</td>
<td>75</td>
<td>79.79</td>
<td>63</td>
<td>58</td>
</tr>
<tr>
<td>K</td>
<td>Order / Decimal</td>
<td>68</td>
<td>72.34</td>
<td>51</td>
<td>46</td>
</tr>
<tr>
<td>UCP</td>
<td>Order / Fraction</td>
<td>86</td>
<td>91.49</td>
<td>76</td>
<td>72</td>
</tr>
<tr>
<td>K</td>
<td>Estimate/Fraction</td>
<td>89</td>
<td>94.68</td>
<td>68</td>
<td>81</td>
</tr>
<tr>
<td>UCP</td>
<td>Order / Decimal</td>
<td>89</td>
<td>94.68</td>
<td>63</td>
<td>70</td>
</tr>
<tr>
<td>K</td>
<td>Order / Fraction</td>
<td>70</td>
<td>74.47</td>
<td>62</td>
<td>50</td>
</tr>
<tr>
<td>K</td>
<td>Pictorial / Fraction</td>
<td>87</td>
<td>92.55</td>
<td>86</td>
<td>68</td>
</tr>
<tr>
<td>URP</td>
<td>Fraction Division</td>
<td>48</td>
<td>51.06</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td>URP</td>
<td>Decimal Division</td>
<td>53</td>
<td>56.38</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>K</td>
<td>Reading Decimal</td>
<td>69</td>
<td>73.40</td>
<td>80</td>
<td>65</td>
</tr>
<tr>
<td>UCP</td>
<td>Pictorial Estimating</td>
<td>55</td>
<td>58.51</td>
<td>57</td>
<td>40</td>
</tr>
<tr>
<td>UCP</td>
<td>Pictorial Estimating</td>
<td>75</td>
<td>79.79</td>
<td>72</td>
<td>75</td>
</tr>
<tr>
<td>URP</td>
<td>Fraction Subtraction</td>
<td>77</td>
<td>81.91</td>
<td>55</td>
<td>52</td>
</tr>
<tr>
<td>K</td>
<td>Equivalence</td>
<td>75</td>
<td>79.79</td>
<td>63</td>
<td>61</td>
</tr>
<tr>
<td>K</td>
<td>Equivalence</td>
<td>59</td>
<td>62.77</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>I&amp;SP</td>
<td>Fraction Addition</td>
<td>70</td>
<td>74.47</td>
<td>52</td>
<td>45</td>
</tr>
<tr>
<td>URP</td>
<td>Decimal to Fraction</td>
<td>51</td>
<td>54.26</td>
<td>46</td>
<td>36</td>
</tr>
<tr>
<td>URP</td>
<td>Decimal Subtraction</td>
<td>80</td>
<td>85.11</td>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>UCP</td>
<td>Proportion</td>
<td>42</td>
<td>44.68</td>
<td>44</td>
<td>52</td>
</tr>
<tr>
<td>I&amp;SP</td>
<td>Fraction Multiplication and whole number subtraction</td>
<td>34</td>
<td>36.17</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>I&amp;SP</td>
<td>Decimal Multiplication</td>
<td>70</td>
<td>74.47</td>
<td>62</td>
<td>54</td>
</tr>
<tr>
<td>I&amp;SP</td>
<td>Ratio</td>
<td>76</td>
<td>80.95</td>
<td>55</td>
<td>45</td>
</tr>
</tbody>
</table>

All the questions in the cognitive domain of “knowing” are listed in Table 4.4. Participants should not need to perform any type of operation to answer these questions. As you can see from the table, the percent of students who answered correctly was above 80% on only three of the seven questions.
Table 4.4: Part I of Pretest – Scores for Cognitive Domain of Knowing

<table>
<thead>
<tr>
<th>Cognitive Domain</th>
<th>Question</th>
<th>Number of Correct Responses out of 94</th>
<th>Percentage Answering Correctly</th>
<th>8th graders US</th>
<th>8th graders International</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Ordering Decimals</td>
<td>68</td>
<td>72.34</td>
<td>51</td>
<td>46</td>
</tr>
<tr>
<td>K</td>
<td>Number Estimate of Point P on a Number Line</td>
<td>89</td>
<td>94.68</td>
<td>68</td>
<td>81</td>
</tr>
<tr>
<td>K</td>
<td>Ordering Fractions</td>
<td>70</td>
<td>74.47</td>
<td>62</td>
<td>50</td>
</tr>
<tr>
<td>K</td>
<td>Pictorial Representation of Fraction</td>
<td>87</td>
<td>92.55</td>
<td>86</td>
<td>68</td>
</tr>
<tr>
<td>K</td>
<td>Written Form of a Decimal</td>
<td>69</td>
<td>73.40</td>
<td>80</td>
<td>65</td>
</tr>
<tr>
<td>K</td>
<td>Equivalence</td>
<td>75</td>
<td>79.79</td>
<td>63</td>
<td>61</td>
</tr>
<tr>
<td>K</td>
<td>Equivalence</td>
<td>59</td>
<td>62.77</td>
<td>49</td>
<td>49</td>
</tr>
</tbody>
</table>

Only 72.3% and 74.5% of the preservice teachers correctly ordered decimals and fractions, respectively. In the decimal number problem, students were asked to identify the smallest number from a list of decimal numbers. The correct answer in this problem was 0.125 but 21 of the participants answered that the largest number (0.625) was the smallest. Since there was no work or explanation to accompany this problem, it is unclear whether the students did not read the problem carefully or if they misunderstand place value. The ordering fraction problem was related to a pictorial representation. The students were given the picture in Figure 1.1 and asked what fraction of the circle was shaded. Seventy chose the correct range \( \left( \frac{1}{2} \text{ and } \frac{3}{4} \right) \) but 19 chose the range \( \frac{3}{4} \text{ and } 1 \) and 5 chose the range \( \frac{1}{4} \text{ and } \frac{1}{2} \).
Only 73.4% identified what two hundred six and nine-tenths was from a list of decimal numerals. Of those who answered incorrectly, the most common mistake was identifying this number as 206.09, indicating that there was a lack of knowledge regarding place value.

In this cognitive domain, two questions concerned fraction equivalence. In the first equivalence question, 79.8% correctly picked a group of three equivalent fractions from 4 groups. The second equivalence problem required more conceptual knowledge. The participants were asked to shade $\frac{3}{8}$ on a 4x6 grid. The difficulty was that there were 24 unit squares. Only 62.8% were successful at this task. Three left the region blank. One outlined 8 of the blocks and shaded three and one drew an altogether separate figure of 8 blocks and shaded three. Of the 30 other incorrect responses, 7 participants shaded three blocks, 8 participants shaded 8 blocks, and the other 15 participants shaded various other incorrect responses. This indicates a breakdown with basic fraction knowledge. Further investigation shows that 9 participants missed both of these equivalence questions. Thirty-eight missed only one of the questions, leaving only 47 participants, or 50%, that answered both questions on equivalence correctly.
On a more positive note, there were two questions in the cognitive domain of knowing that students did well on overall. The majority correctly estimated a number that corresponds to a point on the number line and identified a picture that represents the fraction $\frac{2}{3}$. Six students did not identify the picture that showed $\frac{2}{3}$ of a square shaded. They identified squares that were clearly not divided into regions of equal size.

Table 4.5 is the breakdown for problems from the cognitive domain of “using routine procedures.” These problems required the participants to use a routine procedure to solve a problem that had already been set up and was not part of an application problem.

<table>
<thead>
<tr>
<th>Cognitive Domain</th>
<th>Question</th>
<th>Number of Correct Responses out of 94</th>
<th>Percentage Answering Correctly</th>
<th>8th graders US</th>
<th>8th graders International</th>
</tr>
</thead>
<tbody>
<tr>
<td>URP</td>
<td>Fraction Division</td>
<td>48</td>
<td>51.06</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td>URP</td>
<td>Decimal Division</td>
<td>53</td>
<td>56.38</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>URP</td>
<td>Fraction Subtraction</td>
<td>77</td>
<td>81.91</td>
<td>55</td>
<td>52</td>
</tr>
<tr>
<td>URP</td>
<td>Decimal to Fraction</td>
<td>51</td>
<td>54.26</td>
<td>46</td>
<td>36</td>
</tr>
<tr>
<td>URP</td>
<td>Decimal Subtraction</td>
<td>80</td>
<td>85.11</td>
<td>77</td>
<td>77</td>
</tr>
</tbody>
</table>

In three of the five questions in this category more than 43% of the 94 participants answered the questions incorrectly. Two of the problems dealt with the operation of division. On the first question, students were asked to compute $\frac{6}{55} \div \frac{3}{25}$. Forty-eight out of 94 (51%) completed this task correctly. Of the 46 that answered incorrectly, 17 showed no work at all. Since the problem was number 9 on the assessment, there is little
reason to think that they just ran out of time. Two did end up with the correct answers, but simplified incorrectly so their answers were considered incorrect. Thirteen answered the problem correctly but did not give their answer in simplified form; these students were considered to have answered correctly since the problem did not ask for a simplified answer. Four students had answers that were inverted because they cross-multiplied and did not know the correct placement for the numerator and denominator in the answer. Another student inverted the first fraction so the answer was inverted. Four students inverted the second fraction but multiplied incorrectly and two changed the problem to decimals and got incorrect answers.

Only five more students were successful at dividing decimals. There were five choices for the answer to the problem, $0.003\overline{1}5.45$ and the 39 incorrect answers were split among the four incorrect responses. The incorrect answer that was given most often occurred because the students failed to move the decimal in the dividend.

Another problem that students had significant difficulty with was changing 0.48 to a fraction in lowest terms; only 54.3% correctly completed this task. This problem was not multiple choice so students could not guess at an answer or work backwards from the given choices. Of the 43 participants who answered incorrectly, 12 left the problem blank. Three placed the wrong power of 10 in the denominator and 8 either simplified incorrectly or not at all. Four placed the 48 in the denominator, three gave whole number or mixed number responses, 7 gave some version of $\frac{1}{2}$ and 6 gave random fractions with no supporting work.

The next cognitive domain is “using complex procedures.” Table 4.6 shows the results for this cognitive domain.
Some of the participants once again encountered difficulty with equivalence. They had to choose a picture that showed that \( \frac{2}{5} \) is equivalent to \( \frac{4}{10} \). In order to answer this question it was necessary to not only understand equivalence but also to recognize multiple ways of looking at a picture. The correct picture representation for this problem can be found in Figure 4.2. The 19 participants that gave incorrect answers did not make this connection. The incorrect responses were split between the other three choices indicating that there was not a common misunderstanding among the participants. These choices are shown in Figure 4.3.

![Figure 4.2 Correct Response for Equivalent Fraction Problem](U.S. Department of Education, N.D., pg. 4)
Many of the participants had difficulty with all three of the problems involving ratios and proportions. In the first problem, participants were given a map that was scaled 1 cm to 10 km and they had to find how far apart two towns were on the given map. The percentage of students in this study who answered this correctly was only 58.5% compared to 57% of 8th graders in 1999. In the second problem students had to find the length of a building if the length of a car was 3.5 m long. The car was drawn in front of the building. About 80% answered this question correctly but this was not much better than the 72% of US 8th graders or 75% for the international average. In the third problem involving ratio, participants were asked to find the average weight of a salt crystal if 500 salt crystals weigh 6.5g. Only 44.7% answered this question correctly which is less than 1% better than 8th graders in the US in 1999 and is worse than the international average for 8th graders in 1999. All but four attempted the problem and the majority of the 48 incorrect responses were split between the answers 0.0325g (19 participants) and 0.078g (24 participants). Seventeen of the 28 participants who answered 0.0078g or 0.078g, had
work that showed they divided incorrectly and had a seven in their answer so they may have picked one of these solutions based on similarities.

There were two problems in this category that participants did well on and they both dealt with ordering numbers. More than 90% of the participants answered each of these problems. In the first problem, students had to choose the smallest fraction from a list of four. Three of the fractions were unit fractions with the largest being \( \frac{1}{2} \). The fourth choice was \( \frac{2}{3} \) so that when examining the choices students needed only to look at the largest denominator to pick the smallest fraction. Five chose the incorrect answer, apparently thinking the smaller the denominator in a unit fraction the smaller the fraction. In the second problem, students were asked to identify a number, from four choices, between 0.07 and 0.08. Only 5 missed this problem indicating a possible misunderstanding with place value. Only 1% missed both problems and 86% answered both questions correctly indicating that most students have an overall good grasp of basic concepts involving ordering fractions and decimals.

Participants were not as successful with the two problems from the cognitive domain knowing that dealt with numerical order and place value. One of those problems was to pick the smallest decimal and only 72% did this correctly. When given a circle with a fraction of the circle shaded only 74% correctly chose the range of the shaded portion.

Table 4.7 shows the results from the cognitive domain “investigating and solving problems.” These problems required basic knowledge of fractions but also required problem-solving skills. On four of the five problems, fewer than 75% of respondents
answered correctly and on the problem involving multiple operations, only 36% answered correctly.

<table>
<thead>
<tr>
<th>Cognitive Domain</th>
<th>Question</th>
<th>Number of Correct Responses out of 94</th>
<th>Percentage Answering Correctly</th>
<th>8th graders US</th>
<th>8th graders International</th>
</tr>
</thead>
<tbody>
<tr>
<td>I&amp;SP</td>
<td>Proportion</td>
<td>68</td>
<td>72.34</td>
<td>68</td>
<td>69</td>
</tr>
<tr>
<td>I&amp;SP</td>
<td>Fraction Addition and Subtraction</td>
<td>70</td>
<td>74.47</td>
<td>52</td>
<td>45</td>
</tr>
<tr>
<td>I&amp;SP</td>
<td>Fraction Multiplication and whole number subtraction</td>
<td>34</td>
<td>36.17</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>I&amp;SP</td>
<td>Decimal Multiplication</td>
<td>70</td>
<td>74.47</td>
<td>62</td>
<td>54</td>
</tr>
<tr>
<td>I&amp;SP</td>
<td>Ratio</td>
<td>76</td>
<td>80.95</td>
<td>55</td>
<td>45</td>
</tr>
</tbody>
</table>

In this cognitive domain, participants were most successful on a question that asked them to find the ratio of nitrate to the total amount of fertilizer when given all the ingredients. The most frequent incorrect response was the ratio of nitrate to all other ingredients, demonstrating that participants did not know to include the nitrate as part of the whole. In the problem involving proportion, 72.3% of the participants answered correctly. That left 26 participants that did not give the correct response to the problem: If there are 300 calories in 100g of a certain food, how many calories are there in a 30g portion of this food? (U.S. Department of Education, N.D., pg. 2)

The three problems that required fraction operations resulted in less success. In two of these problems, 74.5% of participants answered correctly. The first question was:

Robin and Jim took cherries from a basket. Robin took \(\frac{1}{3}\) of the cherries and Jim
took \( \frac{1}{6} \) of the cherries. What fraction of the cherries remained in the basket?

(U.S. Department of Education, N.D., pg. 25)

Two participants left this problem blank and the other 21 incorrect responses were split among the choices. The second of these two problems required students to find the height of a stack of 400 sheets of paper if one sheet of paper is 0.012 cm thick. The 24 incorrect responses indicated a problem with place value when multiplying decimals. The last problem in this section that had the fewest number of participants answering correctly was:

Laura had $240. She spent \( \frac{5}{8} \) of it. How much money did she have left? (U.S. Department of Education, N.D., pg. 30)

Only 36.2% of participants were able to correctly answer this question. Sixty participants missed this problem and 22 of these showed little or no work and left the answer blank. Thirteen participants answered the wrong question; they stopped once they multiplied \( \frac{5}{8} \) and $240. The other 15 incorrect responses consisted of 20 different responses and had flaws in their work that indicated a lack of understanding.

The results from this part of the assessment indicate that there may be a serious gap in what preservice teachers know and understand and what they are expected to know as they enter into their mathematics content courses here at Southeastern University. This has serious implications in regard to what should be taught and how it should be taught. These implications will be discussed in Chapter Five.
Assessment, Part 2

In Part 2 of this assessment, participants were asked to solve two different problems two different ways. Both problems were application problems on which students had to determine what operation was needed and then solve them both by drawing a picture and then by using an algorithm. The first problem was taken from Elementary and Middle School Mathematics: Teaching Developmentally by Van De Walle.

Paul and his brother were each eating the same kind of candy bar. Paul had \( \frac{3}{4} \) of his candy bar left. His brother still had \( \frac{7}{8} \) of his candy bar. How much candy did the two boys have together? (Van de Walle, 2001, pg. 229)

While this is an application problem that requires the ability to set up and solve the problem, I did not anticipate that students would have difficulty with it. However, only 37% were able to set this problem up correctly and reach a complete answer. A complete answer was a mixed number (without units). If an answer was left as an improper fraction, then the answer was considered to be incomplete because of the context of the problem – 17% fell into this category. This leaves 46%, or 43 participants, that did not solve the problem or reached an incorrect answer. Of these 43 participants, 11 did not set the problem up at all. One participant had the correct answer but had no supporting work. The other 31 were able to set the corresponding addition problem up but had flawed work. The breakdown of the errors is as follows:

- Nine (9.5%) found a common denominator but added both numerators and denominators, another four (4.2%) found a common denominator but
made another error with addition or performed the wrong operation, and one more made a mistake finding a common denominator and then did not add.

Six (6.3%) did not find a common denominator but added both numerators and denominators and one (1%) did not find a common denominator and multiplied numerators and denominators.

Two (2.1%) attempted to get a common denominator but failed and did nothing else and one found a common denominator but did nothing else.

Seven (7.4%) answered the wrong question (how much they ate).

Solving this problem using a picture posed more difficulty. Complicating this process even further, when solving the problem using this method students were asked to explain the solution process so that any drawing they had would make sense. One possible correct response would be to represent \( \frac{3}{4} \) and \( \frac{7}{8} \) in individual pictures and then show or explain how \( \frac{3}{4} \) equals \( \frac{6}{8} \) and why it was necessary to change \( \frac{3}{4} \) to an equivalent fraction of \( \frac{6}{8} \). This would then be followed by combining the two pictures to show an answer of \( 1 \frac{5}{8} \) with an explanation accompanying this step as well. Not a single participant solved this problem with pictures and explained the process completely; however, all but 6 at least drew a picture to start the process. Sixteen participants made a mistake in drawing the initial fractions. Ten of these 15 drew different size units to represent the two candy bars. These ten pictorial representations all implied that \( \frac{1}{4} = \frac{1}{8} \).
Figure 4.4 is an example of a young lady whose initial pictures showed this common mistake. Notice that she did not come up with the correct response when drawing a picture.

![Figure 4.4 Different Size Units to Represent Same Size Candy Bar](image)

Five made other errors with shading and one even represented \( \frac{7}{8} < \frac{3}{4} \) with her drawing.

Of the 73 participants who drew the initial fractions without making a mistake, 55 drew the initial fractions and did not complete the process by combining the two drawings to get a picture that shows the answer. The following is a description of these responses with participant examples:

- 27 showed the initial pictures making no reference to \( \frac{3}{4} = \frac{6}{8} \)
  - 7 gave correct answers
  - 5 gave incorrect answers
  - 1 answered the wrong problem
14 gave no answer

15 showed the initial pictures making some reference to $\frac{3}{4} = \frac{6}{8}$ but not showing the process with a picture

7 gave correct answers

4 gave incorrect answers

1 answered the wrong problem

3 gave no answer

Figure 4.4 shows work that was from one of the middle grades education majors. He made the connection that $\frac{3}{4} = \frac{6}{8}$ but did not show the process. He did however try to explain that he changed $\frac{3}{4}$ to a similar amount as $\frac{7}{8}$ but does not elaborate on what he means and his lack of mathematical terminology can be confusing. He also failed to give a final answer for this problem when using this method.

Figure 4.5 Middle Grades Education Major Pictorial Solution for Addition Problem
13 showed the initial pictures showing the process of \( \frac{3}{4} = \frac{6}{8} \) with a picture

- 6 gave correct answers

- Figure 4.6 is an example of a response in which the student showed and explained that \( \frac{3}{4} = \frac{6}{8} \). She also explained how she came up with her answer even though she did not use a final picture to come up with the solution.

![Figure 4.6 Participant’s response with Good Connection for Common Denominator](image)

- 3 gave incorrect answers

- Figure 4.7 is an example of work that was done by a student who has taken this course before. Notice that she does a good job showing why \( \frac{3}{4} = \frac{6}{8} \) but does not show how to combine \( \frac{6}{8} \) and
\frac{7}{8} with a picture. She then makes a mistake with her addition, which was the same mistake she made when solving this using an algorithm. It is unclear from what she wrote whether she did indeed add incorrectly or if she was viewing the unit as 2 candy bars which would give 16 pieces in the unit instead of 8.

Figure 4.7 A Repeater Error with Addition

- 1 answered the wrong problem
- 3 gave no answer
- 18 showed initial pictures and a correct final picture
- 13 drew the initial fractions in some format with an addition sign between them and said that was equal to another drawing. These 13 had no explanation at all, though their work shows an understanding of the problem and its solution.
The student work in Figure 4.8 demonstrates this. Some of the others included an answer written next to the final drawing.

Figure 4.8 Initial and Final Fractions as Picture with No Explanation

Figure 4.9 is an example of another participant who shows drawing the process. She did not make a connection verbally that \( \frac{2}{8} = \frac{1}{4} \) but the connection can be seen in her work.

Figure 4.9 Connecting \( \frac{2}{8} = \frac{1}{4} \) without Verbal Explanation
3 had pictures drawn for each stage of the process but had errors that led to an incorrect solution.

Figure 4.10 is an example of student work that shows pictures for the initial fractions and an incorrect answer that indicates the student approached the problem using multiplication.

![Figure 4.10 Trying to Approach Addition Problem as Multiplication](image)

Figure 4.11 shows an example of student work that exemplifies how a student showed that \( \frac{3}{4} = \frac{6}{8} \), but viewed the final answer in terms of \( 16^{\text{th}} \).
There were two that offered explanations with their drawings. The better of these two solutions is shown in Figure 4.12. This student was the only participant that looked at the candy bar in pieces when she combined them.
The second question dealt with division of fractions and again students were asked to solve this problem in two ways. Because this was expected to be more difficult for students to represent with a picture, two different questions were given to the participants to see if one type of question was easier to solve than the other. The first question was a whole number divided by a fraction that ended up with a whole number answer and is as follows:

Megan is making a necklace that will be 16 inches long. To make the necklace she strings a thin wire with \( \frac{3}{2} \) inch beads. How many beads will she need to make the necklace?

Fifteen out of 48 people set this division problem up and used the standard algorithm to solve it correctly, two had a correct answer with no supporting work, one set up the problem correctly but made a multiplication error after inverting, and one set it up as a division problem but did not follow through. An additional person set the problem up correctly but encountered difficulty multiplying the fractions after inverting and one set the problem up as \( \frac{2}{3} \div 16 \). One participant set the problem up two different ways and scratched out the correct solution and answer. Three people set up the equation \( \frac{2}{3} x = 16 \) but only one got the correct answer and recognized it as such. Two people recognized how this problem could be solved using a repeated addition approach; however, one added up to only ten inches and the other person made several errors in his addition which can be seen in Figure 4.13. The next figure, Figure 4.14, shows a participant’s work where the problem was set up backwards but she still managed to get to the correct answer. There were seven that did nothing to set this problem up and
another six that had other work shown on their paper that was not working towards a correct answer. Two had an incorrect answer on their papers with no supporting work, four approached this as multiplication, and two approached this as multiplication but as $\frac{2}{3} \times \frac{1}{16}$. While the participants provided no explanation for multiplying by $\frac{1}{16}$, one possibility is that they were initially thinking of the problem as $\frac{2}{3} \div 16$.

Figure 4.13 Repeated Addition Approach

![Repeated Addition Approach](image)

Figure 4.14 Problem set up backwards but correct answer

![Problem set up backwards but correct answer](image)
The other problem was a little more difficult since it was a mixed number divided by a fraction that ended up with a mixed number as an answer. The problem was:

John is building a patio. Each section requires \( \frac{2}{3} \) of a cubic yard of concrete. The concrete truck holds \( 2 \frac{1}{4} \) cubic yards of concrete. If there is not enough for a full section of concrete at the end, John can put in a divider and make a partial section. How many sections can John make with the concrete in the truck? (Van de Walle, 2001, pg. 239)

Based on the results, this problem was more difficult for the participants than the previous problem. Only five of the 46 participants could completely answer this problem with procedural knowledge. The responses are discussed below.

- Two set the problem up as division, found a common denominator but did not divide.

- Three set up the problem correctly but instead of leaving the answer as an improper fraction or changing the number to a mixed number all three used long division to change the improper fraction to a decimal and did not get an exact answer.

- Four set the problem up correctly, found common denominators and got \( \frac{3}{12} \) as an answer instead of \( \frac{3}{8} \).
Three set the problem up as \[ \frac{2}{3} \div \frac{9}{4} \]. After working this problem out and getting an answer of \[ \frac{8}{27} \], one participant then set the problem up correctly but stated, “Don’t know how to divide \( 2 \frac{1}{4} \) by \( \frac{2}{3} \) to get a whole number answer.”

Three tried the problem as either repeated addition or repeated subtraction but were unable to get the correct answer. The one that was the closest is shown in figure 4.15.

![Division Approached as Repeated Addition](image)

Figure 4.15 Division Approached as Repeated Addition

Two set the problem up correctly but made a mistake in changing \( 2 \frac{1}{4} \) to an improper fraction.

Of those that did not recognize the problem as division, two found a common denominator and gave an answer of 3 without supporting work, one gave a common denominator and nothing else, one incorrectly changed \( 2 \frac{1}{4} \) to an improper fraction and...
found a common denominator, two just changed $\frac{21}{4}$ to an improper fraction, two set the problem up as addition, two set the problem up as subtraction, ten did nothing and four that had random work that did not lead to a correct answer.

The mean scores of the two division problems were lower than the means of the addition problem when the participants were asked to draw a picture and explain how the picture shows the solution. Out of the 48 people who worked the necklace problem, ten people drew no picture. Twenty-three drew either a necklace, number line, or several beads to represent the length of the necklace with no other work or answer shown. Four had pictures but wrote the same incorrect answer that they showed with the algorithmic solution. Two more showed a picture of a necklace and reworked the problem with an algorithm but had the correct answer. One participant appears to have made an assumption that you could only put one bead per inch; therefore, she got an answer of 15 beads with her drawing. This can be seen in Figure 4.16.

Figure 4.16 Incorrect Assumption of One Bead per Inch
There were nine that made a connection with how to use a picture to solve this problem. Nevertheless, not all of these nine participants relied on a picture to get the solution. Figure 4.16 shows the work of a participant who found a common denominator between 16 and \( \frac{2}{3} \) and made a good connection with circles, but did not follow through with a drawing and she did not reach an answer.

![Figure 4.17 Good Connection to Picture with No Follow Through](image)

Three were on the right track with their drawings but made an error in the process. The first of these three participants drew beads on a number line and numbered below the beads in increments of \( \frac{2}{3} \) up to 16. He made his first mistake right after four inches which can be seen in Figure 4.18. When asked to solve this with an algorithm, he approached the problem as a repeated addition problem and made the same mistake that he made with his drawing. The main difference was that he reached an answer of 19 beads with his repeated addition approach but in his drawing he reached an answer of 18 beads. He had
19 beads drawn originally but erased that 19\textsuperscript{th} bead when he reached 16 inches. It is unclear whether he caught this discrepancy. Figure 4.19 shows how a student used the process that 3 beads fit in two inches but made a mistake along the way and she acknowledges this.

Figure 4.18 Mislabeling of Inches

Figure 4.19 Student Acknowledging Mistake
The last example where a student that was on the right track made one serious error is shown in Figure 4.20. She stated that “2 beads = \(1\frac{1}{2}\) inch” which is incorrect. This led her drawing to be off, but she had the correct idea about how to use a picture to solve this problem.

![Figure 4.20 Incorrect Assumption](image)

Figure 4.21 is an example from one student who made the connection that there are three beads in two inches and she drew a picture to represent this. Instead of continuing to draw a picture to reach the answer, she used a proportion to get the answer. Figure 4.22 is an example of the work of another participant who made the connection that there were \(1\frac{1}{2}\) beads per inch and drew part of the necklace, then wrote the answer without any other work.
There were two participants who drew a complete picture that showed all the beads and explained the process they used to get to their answers. Both pieces of work have been included in Figure 4.23 and Figure 4.24; both participants employed similar methods to solve this problem when using a drawing.
The results from those who were working the concrete truck problem were even worse than those that worked the necklace problem. Eleven of the 46 drew no picture at all and 32, such as the one shown in Figure 4.25, drew rectangles, trucks, or circles to represent one or more of the initial fractions but did little else. Five of these 20 had incorrect answers with no work or explanation given. There were also more comments written on this section of the assessment from students who did not know how to solve this problem with a picture. Figure 4.26 is an example of one participant who drew a rectangle but nothing else.
There were four that had the basic concept on how to use a picture to solve this problem; however, none of them reached the correct answer. This first example shown in figure 4.27 shows a unique approach for this group of participants. Notice that she viewed each section in thirds, but she made several mistakes that prevented her from reaching the correct answer. Had she drawn $\frac{4}{12}$, then divided that into thirds she would have been closer to a correct answer. The student work in Figure 4.28 is an example where a participant did not draw a picture to get to the answer. Nevertheless, his reasoning along with his diagram shows his understanding and how he could have relied
on a picture alone for the solution. The last two participants better utilized a picture to solve this problem but both encountered a problem figuring out the fractional part of the answer using the picture. Figure 4.29 and 4.30 illustrate their work. Notice that both of these participants said there was \( \frac{3}{12} \) or \( \frac{1}{4} \) of another section to be made which was how much concrete was left in the truck. They did not figure out how much \( \frac{1}{4} \) cubic yard of concrete would fill in a section that requires \( \frac{2}{3} \) cubic yards.

Figure 4.27 Initial Understanding of Using Pictures – Concrete Problem
THE TRUCK CAN HOLD $2\frac{1}{4}$ CUBIC YARDS, WHICH
EQUALS $\frac{9}{4}$ CUBIC YARDS, WHICH EQUALS
$\frac{72}{12}$ CUBIC YARDS. EACH SECTION REQUIRES $\frac{2}{3}$ CUBIC YARDS,
WHICH EQUALS $\frac{8}{12}$ CUBIC YARDS, TO SEE HOW
MANY SECTIONS YOU CAN GET OUT OF $\frac{72}{12}$ CUBIC YARDS
YOU DIVIDE $\frac{72}{12}$ CUBIC YARDS BY $\frac{8}{12}$ CUBIC
YARDS AND GET 3.3... SO YOU CAN MAKE
3 SECTIONS WITH THE CONCRETE.

Figure 4.28 Verbal Understanding – Concrete Problem

---

Figure 4.29 Trouble with Fractional Part – Concrete Problem
On this assessment, the majority of comments were written for drawing a picture on the division problem, indicating more frustration. Two students that were unable to solve the necklace problem at all commented “I’m sorry, I’m not sure how to solve this” and “Lost! No clue!” A third student who could solve the problem using an algorithm stated, “I have no idea how to illustrate this. Sorry!” A student who was unable to solve the concrete problem stated, “Not sure what picture to make.”

**Repeaters**

To determine if there was a noteworthy difference between the scores of those who had previously taken the course and those who had not, I compared their scores. There were 13 participants who had previously taken this course and completed it with a D or F. These 13 were compared to the other 81 who had not taken and finished the course prior to this semester. I included these scores and the scores for the group as a whole in Table 4.8. Ninety-two percent of this sample of participants scored lower than 80% on this part of the assessment. That means that only one of the participants who had
completed this course before this semester scored above 80%. This group of participants answered 17 of the 23 questions with less than 80% accuracy compared with 15 out of 23 questions for those who had not previously taken this course. The repeaters had a higher percentage of participants answering correctly on 7 of the 23 (30%) questions in comparison to group of participants taking the course for the first time.

Table 4.8: Comparison of Repeaters vs. Non-repeaters

<table>
<thead>
<tr>
<th>Cognitive Domain</th>
<th>Question Description</th>
<th>Question Number</th>
<th>Number of Correct Responses out of 13 Repeaters</th>
<th>Percentage Answering Correctly out of 13 Repeaters</th>
<th>Percentage Answering Correctly out of 81 Non-repeaters</th>
<th>Percentage Answering Correctly out of 94 Full Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>I&amp;SP</td>
<td>Proportion</td>
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<td>10</td>
<td>76.92</td>
<td>71.60</td>
<td>72.34</td>
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<td>Equivalence</td>
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<td>11</td>
<td>84.62</td>
<td>79.01</td>
<td>79.79</td>
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<td>K</td>
<td>Order / Decimal</td>
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<td>74.07</td>
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<td>Order / Fraction</td>
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<td>Estimate/Fraction</td>
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<td>76.92</td>
<td>97.53</td>
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<td>Order / Fraction</td>
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<td>38.46</td>
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<tr>
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<td>35.80</td>
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<td>9</td>
<td>69.34</td>
<td>80.72</td>
<td>80.95</td>
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</table>
On Part 2 of the assessment, only four of the thirteen (30.7%) repeaters set up the addition problem correctly and showed sound procedural knowledge. One left that problem blank and the other eight had flawed work. For the pictorial solution, two showed the initial and final picture with no explanation. Eight drew the initial picture of the fractions in the problem but four had problems with their drawings in regard to unit size. The last three of these participants showed that \( \frac{3}{4} = \frac{6}{8} \) with their initial pictures but did not finish the process with pictures and had the incorrect answer with their drawings.

For the division problem only two of the 13 (15.4%) participants showed sound procedural knowledge and they both had the necklace problem. Three of the participants utilized a drawing to solve the problem but only one of these got the correct answer.

These 13 participants had previously taken this course and had been exposed to thinking about mathematics conceptually more recently than those who were taking this course for the first time. The repeaters scored worse on problems that are emphasized in Math I. The implications of this will be discussed in Chapter 5 along with the results of the full sample.

Interviews

There were 11 students that participated in the interview process at the beginning of the semester. Table 4.9 provides the pseudonyms that I will use for these participants, with their major of study, and their general opinion of working with fractions. Only four of the 11 interviewees expressed no opposition to working with fractions and three of these preferred working with fractions over decimals. One of these participants went as far as to say, “I hate decimals! I like fractions better than decimals” (Jane, Interview 9/2005). She stated that she had been working with fractions since she was very young.
because she baked a great deal with her mom and gained hands on experience with common fractions. The other 7 participants were not as fond of working with fractions. When working problems, each thought she would change fractions they encountered to decimals if they felt they were too difficult. Only one admitted that “fractions are a struggle” (Lori, Interview 9/2005). She also said that decimals were not as bad and that she would rather work with them over fractions.

Table 4.9: Participant Information for Interviewees – Fall Semester

<table>
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<tr>
<th>Pseudonym</th>
<th>Major</th>
<th>Opinion of fractions</th>
</tr>
</thead>
<tbody>
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<td>Ali (AO)</td>
<td>Elementary</td>
<td>Depends on difficulty</td>
</tr>
<tr>
<td>Brooks (BB)</td>
<td>Elementary</td>
<td>Likes fractions</td>
</tr>
<tr>
<td>Jane (JJ)</td>
<td>Elementary</td>
<td>Likes fractions, dislikes decimals</td>
</tr>
<tr>
<td>Lindy (LC)</td>
<td>Elementary</td>
<td>Depends on difficulty</td>
</tr>
<tr>
<td>Sandy (SM)</td>
<td>Elementary</td>
<td>Not sure</td>
</tr>
<tr>
<td>Vickie (VC)</td>
<td>Elementary</td>
<td>Likes fractions more than decimals</td>
</tr>
<tr>
<td>Kay (KS)</td>
<td>Exceptional Ed</td>
<td>Not sure, relies on calculator</td>
</tr>
<tr>
<td>Jack (RB)</td>
<td>Exceptional Ed</td>
<td>Depends on difficulty</td>
</tr>
<tr>
<td>Lori (LA)</td>
<td>Exceptional Ed</td>
<td>Struggles with them</td>
</tr>
<tr>
<td>Sue (SB)</td>
<td>Exceptional Ed</td>
<td>Not sure</td>
</tr>
<tr>
<td>Tory (TD)</td>
<td>Middle Grades</td>
<td>Likes fractions, thinks decimals are harder</td>
</tr>
</tbody>
</table>
Understanding of a Fraction

To better understand how participants perceived fractions, they were asked to talk about the fraction $\frac{2}{3}$ and tell me everything that came to mind. This appeared to be a difficult task for the participants. Part of the difficulty could have stemmed from the participants’ uneasiness with talking about mathematics, the openness of the question, or a lack of knowledge. In an attempt to get at each participant’s understanding of what a fraction is and how it can be used, I had to occasionally asked leading questions.

Eight of the 11 interviewees viewed fractions as parts of a whole and drew a picture using circles or rectangles. Sandy also immediately connected $\frac{2}{3}$ to a decimal and then drew $\frac{2}{3}$ using a circle. Six of these eight participants thought of a fraction in another way, such as parts of a collection, but only when asked leading questions. Jane immediately connected this concept to ratios and having 2 out of 3 objects. Jane and Kay were the only two interviewees that immediately thought of fractions as parts of collections but not as parts of a whole. Even though they thought of fractions as parts of a collection, they did not think of fractions as ratios. Lindy thought ratios were different than fractions all together. She mentioned that she thinks of a fraction as a ratio but sounded really unsure of herself and then said they were different. I then gave her an example using 2 out of 3 boys. Lindy then said that was a fraction. I asked if it was a ratio and she said yes, but “you just write if different” (Lindy, Interview 9/20005). When I asked how, she said with a colon. Vickie was the only participant to immediately give an example of a fraction using an example with both parts of a whole and parts of a collection.
These responses show that this group of participants, as a whole, had a very unsophisticated understanding of fractions. Most of these eleven participants viewed fractions only as parts of a whole and they gave examples of $\frac{2}{3}$ using pizza or pie.

Only one participant’s understanding of a fraction encompassed more than one conception without being asked leading questions. The implications this could have on the students these participants might one day teach will be discussed in Chapter 5.

*Fraction Addition*

The next topic we discussed in the interviews was addition of fractions. I gave each interviewee two addition problems one with common denominators and one without. Ten of the 11 participants noticed the difference in the two problems and the difference in how to solve each of them algorithmically. Nine of these ten computed both of these addition problems with little or no problem. Lori was the one interviewee that noticed the differences but did not successfully solve the problem where a common denominator was needed to find the sum. While she successfully added the two fractions that already had like denominators, she said she would rely on a calculator to change the fractions to decimals and then add them. I asked leading questions that helped her reach an answer. Jack was not sure if the problems were different. So when he solved them, he solved them the same way by adding numerators and denominators together. He then corrected himself and decided that if the denominators were not the same he did need to find a common denominator first. The problem was $\frac{3}{4} + \frac{2}{3}$ and he made a critical error when finding the common denominator. He added the denominators together giving him 7. He then multiplied each numerator by 3 to come up with what he thought was the
equivalent problem of $\frac{9}{7} + \frac{6}{7}$. He was certain that he was correct until I pointed out that 12 was the common denominator. He was then able to finish the problem and reach the correct answer.

Since these addition problems were not in context, I compared how they answered questions in the interview with what they did on the assessment given in the first week of class. Nine had solved the addition problems without difficulty and eight had solved the candy bar problem correctly on the assessment administered at the beginning of the semester. Tory did not solve the addition problem correctly when in context. She set it up correctly but then added numerators and denominators. Lori and Jack were also unsuccessful at completing an addition problem in context.

During each interview, I then asked interviewees to use pictures to solve these problems. None of the participants used pictures to reach the sum of $\frac{3}{4} + \frac{2}{3}$. Each one could represent the initial fractions in some fashion but using the pictures to find a common denominator was problematic. Seven utilized a circle model to represent each fraction and then encountered problems with dividing the regions properly. Kay used squares and encountered the same problem. Two used the concept of parts of a collection to represent the fractions and one of these realized the difficulty with this representation. So she tried using circles and rectangles without success. Jane tried to approach this problem by viewing the fractions as parts of a collection. However, when she combined these she was unable to reach an answer and could not figure out where she went wrong. Her work is shown in Figure 4.31. Notice that she viewed each fraction as a part of a collection. Then when she combined them, she did so as if they were ratios. Her last step
shows 17 over 12 which is the correct answer but notice she is matching up items as if she were going to cancel them. At this point she communicated her confusion regarding solving the problem using pictures. This lack of success could be due in part to the lack of context. To see if these 11 participants had greater success when working an addition problem in context, I compared these results to their pretest results.

![Figure 4.31: Jane’s Approach to Addition](image)

All of the participants, including Jane, used either a circle or rectangle to draw pictures that represented the candy bars from the addition problem in the pretest. Brooks was the one participant who did a good job on the pretest utilizing a picture to solve the candy bar problem, but she did not experience the same success during the interview. There is the possibility that the lack of context posed a problem.

Now that I knew most of the interviewees could add fractions that were not in the context of word problem, I wanted to know if any of participants could explain why a
common denominator is needed when adding fractions with the standard algorithm. Seven had no explanation at all for this process. Jane used an analogy to fruit and another used an analogy to last names. Another interviewee said “I don’t know the actual reason why. I have just always been told to do it when adding and subtracting” (Sandy, Interview 9/2005). Two participants said it was harder to add things that were different sizes. When asked to elaborate, one used unit fractions to explain that $\frac{1}{3}$ was bigger than $\frac{1}{4}$.

All of the participants agreed that you could not add fractions unless there was a common denominator. When given simple examples there were only 3 participants who quickly computed the sum. Two said they actually got a common denominator in their heads and then add them and one related the fractions to money. Yet another participant, when talking about the problem $\frac{1}{2} + \frac{3}{4}$, said “I think you could add them because 2 and 4 are powers of each other” (Jack, Interview 9/2005). He noticed that somehow the denominators were related but was not sure how this helped. He also did not get the correct answer. It is important to note he was also the one participant who did not solve the previous addition problem. The last interviewee said that for the second problem involving one-half and three-fourths, she would visualize pictures to get to the answer, “like cookies” (Sue, Interview 9/2005).

**Fraction Division**

In each interview I moved on to division and asked each participant to divide a mixed number by a fraction ($2 \frac{1}{4} \div \frac{2}{3}$). Eight participants were able to complete this
problem successfully without any help. Sandy was the only one of these 8 participants that found a common denominator before dividing. She was unsure what to do next. After talking, she computed the problem without common denominators. Lori and Sue completed this task but only with a big hint from me. I asked each of them if I told them multiplication was involved if that would help and that was enough for them both to complete the problem. Ali was the only participant who did not complete this problem at all. Again since this problem was not in context, I compared these results with those from the participants’ pretest. When presented with an application problem in context, only 4 of these participants successfully set-up and solved the problem. One found a common denominator and then estimated the answer. The other 7 did not successfully complete this problem.

Using the same division problem, $\frac{3}{2} \div \frac{4}{12}$, I asked each interviewee to use a picture to solve this problem. Not one of the participants could do this. I then changed the problem to a whole number divided by a fraction ($4 \div \frac{2}{3}$) and again no one could solve this problem with a picture. However, Jane once again approached the problem differently than the other participants. Instead of drawing 4 wholes, she drew 4 sets of 3. She was making progress on solving the problem but became frustrated trying to figure out what to do with the un-shaded shapes. Her work can be seen in Figure 4.32.
To get at their understanding of division and how division is used in models, I then gave a problem involving only whole numbers (6 ÷ 3). Four did not represent this pictorially but said they understood once I led them through the process. However, they did not connect this understanding back to either of the previous problems. The other 7 had no trouble with a pictorial representation of whole number division. However, four needed assistance with 4 ÷ 2/3 and three were able to complete this problem pictorially with no problem. Only one was almost successful in representing the solution to $2\frac{1}{4} \div \frac{2}{3}$ but encountered difficulty with the fractional part of the answer. I again compared these results to the pretest and found that Tory was the only one to successfully take the division problem from her pretest and reach the correct answer using pictures.
At this point in the interview I asked why the division algorithm is to invert and multiply. Not one of the participants could explain this. I encouraged each of them to look at how they solved each one of the problems algorithmically and pictorially to see if they could make a connection. No one ever did. One interviewee said, “I never questioned it” (Brooks, Interview 9/2005). Another participant says she does not remember why, but that she remembers a poem a teacher once taught her, “Turn the divisor awry and multiply” (Ali, Interview 9/2005).

The results of question 1 show that the participants in this study have significant gaps in their fraction knowledge. These gaps exist with procedural knowledge as well as conceptual knowledge and will be discussed in Chapter 5.

Question 2: Knowledge (Experimental vs. Control)

In this section, I relied on quantitative analyses as well as qualitative analysis of pre and post-interviews to answer the following question: Do preservice teachers who have completed an inquiry-based unit possess a better conceptual understanding of fractions and the standard algorithms associated with addition and division than students from a lecture-based unit? These two approaches were discussed in detail in Chapter three. The analysis is divided into two sections. The first section addresses the quantitative analysis which is the analysis of the test measures, including the pre and posttest, given during the semester. The second section is an analysis of the interviews conducted during the semester.
Quantitative Analysis

Participants

One hundred nineteen students agreed to participate in this study at the beginning of the spring semester 2006. However, two dropped during the first week of class and eleven withdrew by the university withdrawal date. Four of these eleven were doing well in the course but had to drop for medical or family reasons. Five participants gave up and stopped coming at some point during the semester after the university withdrawal date. This reduced the possible sample size to 101. The sample size was reduced from 101 to 96 because four participants were absent on quiz day and another participant was removed from this analysis because her ACT score was missing.

Semester Measures

This section looked at assessments conducted during the semester that directly tested fraction knowledge. For this section of analysis a quiz, test, final exam, and difference scores (post-pre) on the pre/post test were used. ACT scores were used as a covariate. There were 49 students in the control group and 47 in the experimental group. Both groups included two sections. The first section in the control group, Group A Lecture had 22 students and the second section, Group B Lecture, had 27 students. The first section of the experimental section, Group A Inquiry, had 22 students and the second section, Group B Inquiry, had 25 students.

Since the $F$ test is robust to non-normality when the non-normality is not caused by outliers, I examined the data to determine if any outliers were present. Only one outlier was found. However, upon further inspection this outlier was considered to be a valid score and it was considered to be a normal part of the group in this educational
setting. Therefore, I determined that it was inappropriate to ignore the outlier and I determined that nonparametric methods were not needed to analyze the data.

I analyzed the data using a multivariate analysis of covariance (MANCOVA) on the dependent variables: quiz, test, final exam, difference (post-pre) scores for both parts of the pre/posttest. The independent variables were instructor and method and the covariate was ACT scores. The results for the MANCOVA results are given in Table 4.10. First I checked for an interaction between instructor and method. With a $p$-value of 0.125 there was insufficient evidence to indicate a significant interaction between instructor and method. Since there was no interaction, I then looked for a main effect for instructor or method. For instructor the $p$-value was 0.530 which is greater than $\alpha=0.1$ so there is also insufficient evidence to indicate that there are any significant differences in test scores as a result of instructor. However, with a $p$-value of 0.035 there is sufficient evidence to indicate that method made a difference in the vector dependent variables.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Test Statistic ($F$)</th>
<th>$p$-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor</td>
<td>0.833</td>
<td>0.530</td>
<td>Not Significant</td>
</tr>
<tr>
<td>Method</td>
<td>2.531</td>
<td>0.035</td>
<td>Significant</td>
</tr>
<tr>
<td>Instructor*Method</td>
<td>1.781</td>
<td>0.125</td>
<td>Not Significant</td>
</tr>
</tbody>
</table>

Since the MANCOVA indicated that there was a significant difference between methods with this vector of test scores, I then examined the individual ANCOVAs. These results are included in Table 4.11. The results show that there was a significant difference
between experimental and control groups (method) for final exam \((p\text{-value} = 0.031)\) scores and for Part 2 of the pre/posttest \((p\text{-value} = 0.022)\). To clarify the direction of the difference by method, the adjusted means have been given in Table 4.12. The results for the MANCOVA suggest that there were significant difference in the vector of scores across methods. In the individual ANCOVAs method was significant for the final exam and Part 2 of the pre/posttest. Since there was a significant difference for method, I looked to see which group performed better on these test measures. On average the experimental group scored 5% better on the final exam and improved an average of 8% more on Part 2 of the pre/post test. These are encouraging results that will be discussed further in Chapter 5.

Table 4.11: Univariate ANCOVA for each Dependent Variable from MANCOVA

<table>
<thead>
<tr>
<th>Variable</th>
<th>Instructor</th>
<th>Method</th>
<th>Instructor*Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test Stat ((F))</td>
<td>(p)-value</td>
<td>Test Stat ((F))</td>
</tr>
<tr>
<td>Quiz</td>
<td>0.412</td>
<td>0.522</td>
<td>0.933</td>
</tr>
<tr>
<td>Test</td>
<td>0.045</td>
<td>0.833</td>
<td>0.028</td>
</tr>
<tr>
<td>Exam</td>
<td>1.562</td>
<td>0.215</td>
<td>4.772</td>
</tr>
<tr>
<td>Part 1 (post-pre)</td>
<td>0.033</td>
<td>0.856</td>
<td>0.002</td>
</tr>
<tr>
<td>Part 2 (post-pre)</td>
<td>1.157</td>
<td>0.285</td>
<td>5.441</td>
</tr>
</tbody>
</table>
Table 4.12: Adjusted Means by Method

<table>
<thead>
<tr>
<th></th>
<th>Control (Lecture)</th>
<th>Experimental (Inquiry)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std Error</td>
</tr>
<tr>
<td>Quiz</td>
<td>75.848</td>
<td>2.517</td>
</tr>
<tr>
<td>Test</td>
<td>77.317</td>
<td>1.824</td>
</tr>
<tr>
<td>Final Exam</td>
<td>71.156</td>
<td>1.643</td>
</tr>
<tr>
<td>Part 1 (post-pre)</td>
<td>3.255</td>
<td>1.512</td>
</tr>
<tr>
<td>Part 2 (post-pre)</td>
<td>43.745</td>
<td>2.533</td>
</tr>
</tbody>
</table>

*Interviews*

*Participants*

At the beginning of the semester there were twenty-four volunteers for the interview process. Two of these participants dropped the class during the first week of the semester, leaving a pool of twenty-two. Thirteen interviews were conducted at the beginning of the semester. One of these participants withdrew from the semester by the University withdrawal date and another did not participate in the end of the semester interview. This leaves data from eleven participants that I examined to see if any change occurred in their fraction knowledge this semester. Table 4.13 contains information regarding the number of interviewees by group and the coding scheme I used to identify them.
Table 4.13: Breakdown of Interview Participants and Codes – Spring Semester

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Method</th>
<th>Code</th>
<th># of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Lecture</td>
<td>AL#</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>Inquiry</td>
<td>AI#</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>Lecture</td>
<td>BL#</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>Inquiry</td>
<td>BI#</td>
<td>2</td>
</tr>
</tbody>
</table>

I will use the codes in Table 4.13 to talk about individual participants so that the section they were in will be identified. There were only two participants in the lecture section with instructor A because only four volunteered originally and one of those dropped the course and another one would not commit to an interview even though she volunteered. In the inquiry section with instructor B there were originally three participants but one withdrew from the course right before the University withdrawal deadline. Following is a brief description of each participant so that subsequent exemplifiers can be put into perspective.

§ AL1 was an elementary education major. She had mostly good experiences in mathematics classes.

§ AL2 was a middle-grades mathematics major. He likes mathematics and has served as the director of a GED program in a neighboring county and taught mathematics in this program. He enjoyed this challenge and decided to return to school to pursue teaching certification.
BL1 was an elementary education major. She had mixed experiences in mathematics classes and attributes this to the quality of the teacher.

BL2 was an elementary education major. She had great experiences until high school.

BL3 was an exceptional education major. He dislikes mathematics and says that it is a struggle.

BL4 was an elementary education major. She is a non-traditional student with a degree in economics and banking experience. She does not hate mathematics but it is not her favorite subject.

AI1 was an elementary education major. She thinks math is intimidating. She already has plans to attend graduate school to get a master’s in exceptional education.

AI2 was an elementary education major. He has taken this class before but had no prior trouble with mathematics classes. He finds it difficult to explain why we do things in mathematics.

AI3 was an elementary education major. Math is not her favorite subject but she does not mind doing it.

BI1 was an elementary education major. Math is not her favorite subject and she gets frustrated with careless mistakes. She has taken this course before but does not think her failure was from lack of understanding but from lack of applying herself.

BI2 was an elementary education major. She only likes some areas of math but stresses the importance of math in everyday life.
Purpose

In the interview analysis I specifically looked for what the participants knew about fractions before and after taking this course and then compared how their knowledge may have evolved differently between methods. The interview questions specifically focused on the participants’ conceptions of what fractions are, their addition and division skills, and their conceptual understanding of the algorithms associated with these two operations.

Understanding of a Fraction

First I looked at the interviewees’ conceptual understanding of fractions. I asked them to explain to me what \( \frac{2}{3} \) meant. At the beginning of the semester, all of the participants gave an example of a part of a whole relationship except for AL2. She also used a collection of objects as an example. The others only drew either a circle or a rectangle to show the fraction. At the end of the semester, three participants in the control group also gave examples of a fraction using a part of a collection relationship and the other three gave only the part of a whole example. In the experimental group the end of the semester brought different results. Only one participant used only the part of a whole relationship. The other four participants gave other examples as well. A11 accidentally started using \( \frac{3}{4} \) and then connected it to \( \frac{75}{100} \) and 75 cents. A12 focused on \( \frac{2}{3} \) and its equivalence to other fractions, such as \( \frac{4}{6} \). In addition to the part to a whole relationship, B11 associated the fraction \( \frac{2}{3} \) to 67%, \( 2 \div 3 \), and 2 out of 3. Lastly, B12 equated \( \frac{2}{3} \) to
0.6 and also a collection of objects to talk about \( \frac{2}{3} \). She does mention that she thought that using a collection of objects was better than using one whole to explain fractions. She then talked about the importance of the unit and being able to recognize what the unit is in a given problem.

_Solving Addition and Division Problems Symbolically and Pictorially_

Next I looked at whether the participants successfully added fractions with and without common denominators using an algorithm or using pictures. Then I asked participants to solve a division problem using an algorithm and picture. These problems differed from Part 2 of the pretest because they were not in context. I have organized the results into Table 4.14 and summarized the results by method in Table 4.15. The “B” (“before”) represents data collected at the beginning of the semester, and the “A” (“after”) represents data that was collected at the end of the semester. A “Y” indicates that the student successfully performed the task without help; a “Y-” indicates the student was successful but needed help, and an “N” indicates that the student did not successfully complete the task.
Table 4.14: Procedural and Pictorial Knowledge Results

<table>
<thead>
<tr>
<th>Participant</th>
<th>Add CD Algorithm</th>
<th>Add CD Pictorial</th>
<th>Add NCD Algorithm</th>
<th>Add NCD Pictorial</th>
<th>Divide Algorithm</th>
<th>Divide Pictorial</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>AL1</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>AL2</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BL1</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BL2</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BL3</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BL4</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>AI1</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>AI2</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>AI3</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BI1</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BI2</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
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</tbody>
</table>
Table 4.15: Summary of Table 4.14 Results by Method

<table>
<thead>
<tr>
<th>Response</th>
<th>Add CD Algorithm</th>
<th>Add CD Pictorial</th>
<th>Add NCD Algorithm</th>
<th>Add NCD Pictorial</th>
<th>Divide Algorithm</th>
<th>Divide Pictorial</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>Lecture</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Y-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Inquiry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Y-</td>
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<tr>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

The table shows that at the beginning of the semester, all of the participants correctly added fractions that already had a common denominator both symbolically and pictorially. In addition, all but one correctly added fractions without a common denominator symbolically without assistance from me. However, BI1 was the only one of the participants that accomplished this pictorially and she had taken this course the previous semester. By the end of the semester all of the interviewees correctly added fractions without a common denominator symbolically and pictorially. At the beginning of the semester, 10 of the participants successfully divided two fractions by using the standard algorithm. One did not complete this task and the last interviewee completed the task with leading questions from me. As for the pictorial representation, BI1 was the only participant that successfully represented the division problem at the beginning on the semester. At the end of the semester only one participant did not represent the problem
pictorially. She started to solve the problem pictorially as one would solve a multiplication problem. She knew that she was wrong and I suggested she think about whole number division to get started. This was enough to refresh her memory and she was then able to solve the problem. Six participants had difficulty with figuring out the fractional part of the answer by looking at a picture and three completed the task without trouble.

*Conceptual Understanding of Algorithms*

The last area that I explored with these participants was their conceptual understanding of the algorithms. At the beginning and end of the semester, interviewees were asked to explain why we get common denominators when adding fractions using the standard algorithm. They were also asked to explain why we invert and multiply when dividing fractions. Table 4.16 shows the before and after results for each participant and Table 4.17 show a summary of results by method. Participants were allowed to utilize pictures and to use written or verbal explanations as desired. An asterisk denotes that a student explained the division algorithm by using complex fractions and the fundamental law of fractions.
Table 4.16: Conceptual Understanding of Algorithms

<table>
<thead>
<tr>
<th>Participant</th>
<th>Addition</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>AL1</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>AL2</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BL1</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BL2</td>
<td>Y-</td>
<td>Y</td>
</tr>
<tr>
<td>BL3</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BL4</td>
<td>Y-</td>
<td>Y-</td>
</tr>
<tr>
<td>AI1</td>
<td>Y-</td>
<td>Y-</td>
</tr>
<tr>
<td>AI2</td>
<td>Y-</td>
<td>Y</td>
</tr>
<tr>
<td>AI3</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>BI1</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>BI2</td>
<td>N</td>
<td>Y</td>
</tr>
</tbody>
</table>

Table 4.17: Summary of Table 4.16 Results by Method

<table>
<thead>
<tr>
<th>Response</th>
<th>Addition</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before</td>
<td>After</td>
</tr>
<tr>
<td>Lecture</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Y-</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Y</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Inquiry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Y-</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Y</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>
In the table a Y- indicated that a participant needed assistance in getting to the correct response. In most cases, these participants lacked appropriate vocabulary or had trouble expressing their thoughts in a concise easy-to-understand sentence. For instance, when BL3 was asked why we get a common denominator when we add fractions using the standard algorithm he said,

Because these are two different fractions. This is three-fourths of a whole and this is two-thirds of a whole. Basically different denominators mean they’re different portions. They’re different size, like portions of a whole. (January 2006)

His response was one of the better ones and it was not incorrect. AI1 encountered a little more difficulty than BL3 did. The following is an excerpt from the beginning of the semester. She had just solved $\frac{3}{4} + \frac{2}{3}$ and explained how she found the common denominator. I then asked why she found a common denominator:

**AI1:** Because the value of three fourths and two thirds isn’t the same. Well,

**Researcher:** When you say the value of them is not the same, what do you mean?

**AI1:** Okay. So three fourths is talking about three of four somethings. I’m not making sense at all.

**Researcher:** You are.

**AI1:** And then two thirds is two parts of three things. So they’re being compared, but one part you have four parts and then in two thirds you only have three parts. So to be able to compare two fractions they have to have the same denominator.

Notice that she is getting close to being able to explain why a common denominator is needed with an algorithm. At the end of the semester, responses were more concise with appropriate language. These responses talked about the denominator as it relates to the size of the piece and that in order to add to fractions the pieces have to be the same size.
The results regarding division of fractions were not very good. In the interviews, it was apparent that few, if any, participants had prepared for the interviews because several made references to how long it had been since they had “studied” fractions. At the beginning of the semester, AL2 was the only participant that was able to provide any justification for inverting and multiplying when dividing fractions. He did have previous experience teaching fractions. His explanation was not conceptual in nature. He used a complex fraction to explain the reasoning behind the algorithm. At the end of the semester only three participants were able to justify the algorithm and all three were from the control group. Two of the three used a complex fraction and one used a more conceptual approach where she used pictures to help explain why we invert and multiply when we divide fractions.

By the end of the semester most of the participants had already demonstrated that they possessed adequate procedural knowledge and that they could utilize pictures to solve addition and division problems. In addition, these results show that most of the participants were able to explain why common denominators are needed when using the standard algorithm to add fractions. However, there was less success when the interviewees were asked to explain why we invert and multiply when we divide fractions. There were only two participants that gave an explanation that was understandable but those explanations did not utilize pictures of fractions and their explanations did not demonstrate conceptual knowledge. These results and their implications will be discussed further in the next chapter.
Questions 3: Beliefs and Attitudes

To look at the data regarding change in beliefs, I analyzed pre and post data for each participant. Three participants failed to fill out the beliefs survey at the end of the semester so they were eliminated from this portion of the study. This leaves a sample size of 98 for this portion of analysis. I will provide the quantitative and qualitative analysis to answer the question: Does an inquiry-based approach improve preservice teachers’ attitudes about mathematics and does it change their beliefs about how they will one day teach mathematics? Once more, the quantitative analysis is divided into two sections. The first section is the analysis of Part I of the survey, whereas the second section will be analysis of Part II of the survey. These two sections will be followed by analysis of the interviews that were performed at the beginning and end of the semester.

Beliefs and Attitudes towards Mathematics

At the beginning and end of the spring semester, participants took a two-part survey. The first part of the survey was a modified version of the Fennema-Sherman Attitude Scales that included 47 questions. Each question was answered by circling a response of A through E. Then each question was scored on a scale from 1 (negative attitude) to 5 (positive attitude); twenty-three of these questions that were reverse coded. Each one of the 47 questions fell into one of four categories:

- Personal Confidence about Mathematics
- Usefulness of Mathematics
- Mathematics as a Male Domain
- Perception of Teacher’s Attitudes
The maximum any participant could score on this survey was 235 points which would indicate a positive attitude towards mathematics.

First I examined the scores for normality among groups. The results of the Shapiro-Wilkes test for normality are included in Table 4.18. Since all four of the groups met the normality assumption at $\alpha = 0.10$, I proceeded with the analysis. In table 4.19, the means for section of the course are included.

Table 4.18: Shapiro-Wilkes Results for Fennema-Sherman

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test Statistic (W)</th>
<th>$p$-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor A - Control</td>
<td>0.935589</td>
<td>0.1444</td>
<td>Normal</td>
</tr>
<tr>
<td>Instructor A – Experimental</td>
<td>0.932192</td>
<td>0.1363</td>
<td>Normal</td>
</tr>
<tr>
<td>Instructor B - Control</td>
<td>0.971186</td>
<td>0.6334</td>
<td>Normal</td>
</tr>
<tr>
<td>Instructor B – Experimental</td>
<td>0.945005</td>
<td>0.1768</td>
<td>Normal</td>
</tr>
</tbody>
</table>

Table 4.19: Fennema-Sherman Means Scores for Pre and Post Survey

<table>
<thead>
<tr>
<th></th>
<th>Lecture</th>
<th>Inquiry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre Mean(std dev)</td>
<td>Post Mean(std dev)</td>
</tr>
<tr>
<td>Instructor A</td>
<td>189.22(21.23)</td>
<td>187.13(22.21)</td>
</tr>
<tr>
<td>Instructor B</td>
<td>179.78(17.56)</td>
<td>186.26(19.78)</td>
</tr>
</tbody>
</table>

I ran repeated measures analysis on the pretest and posttest Fennema-Sherman data with time, instructor, and method as factors. The results are included in Table 4.20.
I first looked at the factor time*instructor*method to see if there was a three-way interaction. There is sufficient evidence (p-value = 0.0561) to indicate an interaction between instructor and method over time.

Table 4.20: Repeated Analysis Results for the Fennema-Sherman

<table>
<thead>
<tr>
<th>Factor</th>
<th>Test Statistic (F)</th>
<th>p-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>5.21</td>
<td>0.0247</td>
<td>Significant</td>
</tr>
<tr>
<td>Time*Method</td>
<td>0.54</td>
<td>0.4636</td>
<td>Not-significant</td>
</tr>
<tr>
<td>Time*Instructor</td>
<td>1.17</td>
<td>0.2823</td>
<td>Not-significant</td>
</tr>
<tr>
<td>Time<em>Instructor</em>Method</td>
<td>3.74</td>
<td>0.0561</td>
<td>Significant</td>
</tr>
<tr>
<td>Method</td>
<td>2.40</td>
<td>0.1244</td>
<td>Not-significant</td>
</tr>
<tr>
<td>Instructor</td>
<td>1.17</td>
<td>0.2824</td>
<td>Not-significant</td>
</tr>
<tr>
<td>Instructor*Method</td>
<td>0.05</td>
<td>0.8295</td>
<td>Not-significant</td>
</tr>
</tbody>
</table>

To aid in interpretation of this three way interaction, I looked at the profile plot for each instructor. These profile plots are shown in Figure 4.33 and Figure 4.34. Notice that the differences in mean scores on the beliefs instrument do not remain constant across methods. The attitudes of the students in instructor A’s inquiry section improved over the semester, whereas the attitudes of the students in instructor A’s lecture section declined over the semester. The attitudes of the students from both of instructor B’s sections improved over the semester. However, the slope of the line for the inquiry group is steeper indicating that there was more improvement in attitudes regarding mathematics in comparison to her lecture-based group. This interaction will be discussed further in Chapter 5.
Figure 4.33: Profile Plot for Instructor A

Figure 4.34: Profile Plot for Instructor B
Beliefs towards Teaching Mathematics

For this section of analysis, I looked at the second part of the survey given at the beginning and end of the semester. This part of the survey consisted of 15 questions. Each question was answered by writing a response of 1, 2, or 3. The responses indicated how much a participant expected their students to exhibit certain behaviors related to learning in their future classrooms. A response of 1 correlated with a teacher-centered classroom and a response of 3 was correlated with a more student-centered classroom. There were 6 questions that were reverse coded. A score of 15 reflected a teacher-centered classroom and a score of 45 reflected a student-centered classroom. The means and standard deviations for each class are included in Table 4.21.

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Method</th>
<th>Pre-Survey</th>
<th>Post-Survey</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std Deviation</td>
<td>Mean</td>
</tr>
<tr>
<td>Instructor A</td>
<td>Lecture</td>
<td>19.30435</td>
<td>3.519323</td>
</tr>
<tr>
<td></td>
<td>Inquiry</td>
<td>18.04545</td>
<td>2.787591</td>
</tr>
<tr>
<td>Instructor B</td>
<td>Lecture</td>
<td>18.37037</td>
<td>3.529223</td>
</tr>
<tr>
<td></td>
<td>Inquiry</td>
<td>18.73077</td>
<td>2.711197</td>
</tr>
</tbody>
</table>

Prior to this analysis, the data were examined to determine if the criteria were met for the assumption of normality. The results of the Shapiro-Wilkes are included in table
4.22. Only one of the two groups met the normality assumption. Further investigation revealed that the data for the control group had two outliers. However, since the outliers were considered to be typical in an educational setting and the $F$ test is robust, I determined that a nonparametric analysis was not necessary.

| Table 4.22: Shapiro-Wilkes Results for Beliefs towards Teaching Mathematics |
|--------------------------|-----------------|----------|------|
| Variable                | Test Statistic ($W$) | $p$-value | Decision   |
| Difference – Control    | 0.941243         | 0.0151   | Not Normal |
| Difference – Experimental | 0.971471        | 0.2887   | Normal     |

I ran repeated measures analysis on the data and the results are included in Table 4.23. First I looked at the interaction time*instructor*method. There was insufficient evidence ($p$-value = 0.1858) that there was not an interaction. Next I looked for a main effect for instructor. At $\alpha = 0.10$, there was no main effect for instructor ($p$-value = 0.4808). However, there was not a main effect for method either ($p$-value = 0.1659). There was a significant difference in overall scores from the pre-survey to post-survey.

To determine if beliefs about teaching moved more towards a student-centered approach or more towards a teacher-centered approach, I looked at the adjusted means that are included in Table 4.24. These means are adjusted since this was an unbalanced design. During the semester for all groups there was an increase in scores indicating that participants would expect their classrooms to be more aligned with a student-centered environment than they expected at the beginning of the semester.
Table 4.23: Repeated Measures Results - Beliefs about Teaching Mathematics

<table>
<thead>
<tr>
<th>Factor</th>
<th>Test Statistic (F)</th>
<th>p-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>16.60</td>
<td>&lt;0.0001</td>
<td>Significant</td>
</tr>
<tr>
<td>Time*Method</td>
<td>1.95</td>
<td>0.1659</td>
<td>Not Significant</td>
</tr>
<tr>
<td>Time*Instructor</td>
<td>0.50</td>
<td>0.4808</td>
<td>Not Significant</td>
</tr>
<tr>
<td>Time<em>Instructor</em>Method</td>
<td>1.78</td>
<td>0.1858</td>
<td>Not Significant</td>
</tr>
<tr>
<td>Method</td>
<td>0.02</td>
<td>0.8754</td>
<td>Not Significant</td>
</tr>
<tr>
<td>Instructor</td>
<td>0.56</td>
<td>0.4577</td>
<td>Not Significant</td>
</tr>
<tr>
<td>Method*Instructor</td>
<td>0.33</td>
<td>0.5697</td>
<td>Not Significant</td>
</tr>
</tbody>
</table>

Table 4.24: Adjusted Means for Each Group - Beliefs about Teaching Mathematics

<table>
<thead>
<tr>
<th>Method</th>
<th>Instructor</th>
<th>Pre Mean</th>
<th>Post Mean</th>
<th>Difference (Post-Pre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inquiry</td>
<td>A</td>
<td>18.045</td>
<td>20.909</td>
<td>2.864</td>
</tr>
<tr>
<td>Lecture</td>
<td>A</td>
<td>19.304</td>
<td>20.087</td>
<td>0.783</td>
</tr>
<tr>
<td>Inquiry</td>
<td>B</td>
<td>18.731</td>
<td>20.038</td>
<td>1.307</td>
</tr>
<tr>
<td>Lecture</td>
<td>B</td>
<td>18.370</td>
<td>19.630</td>
<td>1.26</td>
</tr>
</tbody>
</table>

*Interviews and Journals*

The interview participants, that were introduced in the Interviews section under the heading Question 2: Knowledge (Experimental vs. Control), are the same interviewees included the following analysis. In addition, the codes (AL#, AI#, BL#, BI#) used in that section to identify an interview participant are used in this analysis as well. To facilitate this analysis and present it in an organized fashion, before and after...
tables are used for each group so that change can be detected. The data that are summarized in the table are self reported data that come from interviews and journal entries. A participant’s belief or opinion about a specific topic is coded 1 to 5. Five corresponds to a positive belief or opinion whereas a one is a negative response. Table 4.25 and 4.26 summarize information gathered at the beginning and end of the semester for the control group and experimental groups, respectively. For each area, there are before and after scores. The after scores have a ‘+’ behind the number to denote positive change and a ‘-’ to denote negative change. A brief glance at the tables shows that there was more positive change with the experimental group and more negative change with the control group. To streamline the tables, lengthy headings were omitted from the columns.

$\$ Column 1: General thoughts about mathematics – this includes whether they value mathematics, enjoy mathematics, etc.

$\$ Column 2: Confidence in doing mathematics

$\$ Column 3: Confidence in teaching mathematics

$\$ Column 4: Mathematics content comfort level

$\$ Column 5: Fraction comfort level

$\$ Column 6: Ideal teaching style (1=lecture based, 5=student centered)
Table 4.25: Control Group Self Reported Beliefs

<table>
<thead>
<tr>
<th>Participant</th>
<th>General Thoughts about Mathematics</th>
<th>Confidence in doing Mathematics</th>
<th>Confidence in Teaching Mathematics</th>
<th>Math Content Comfort Level</th>
<th>Fraction Comfort Level</th>
<th>Ideal Teaching Style</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL1</td>
<td>B 5 A 5 1 - B 5 3 - B 2 - A 1 2 + A 1 2 +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AL2</td>
<td>B 5 A 5 5 4 5 + B 3 3 5 3 - A 1 4 +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BL1</td>
<td>B 5 A 5 5 1 5 + B 5 5 3 5 + A 5 4 -</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BL2</td>
<td>B 3 5 + A 1 5 + B 1 4 + B 2 5 + A 5 5 3 4 +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BL3</td>
<td>B 3 5 + A 1 5 + B 2 3 + B 3 2 - A 1 2 + A 1 2 +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BL4</td>
<td>B 5 5 A 3 5 + B 3 5 + B 3 2 - A 3 3 3 4 -</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.26: Experimental Self Reported Beliefs

<table>
<thead>
<tr>
<th>Participant</th>
<th>General Thoughts about Mathematics</th>
<th>Confidence in doing Mathematics</th>
<th>Confidence in Teaching Mathematics</th>
<th>Math Content Comfort Level</th>
<th>Fraction Comfort Level</th>
<th>Ideal Teaching Style</th>
</tr>
</thead>
<tbody>
<tr>
<td>AI1</td>
<td>B 2 5 + A 3 5 + B 3 3 3 3 2 4 + A 3 5 +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AI2</td>
<td>B 4 3 - A 2 3 + B 3 3 3 3 1 5 + A 3 4 +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AI3</td>
<td>B 3 4 + A 3 5 + B 5 5 5 3 - A 1 4 + A 3 5 +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BI1</td>
<td>B 5 3 5 + A 4 5 + B 3 5 + A 2 5 + A 3 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BI2</td>
<td>B 4 5 + A 3 4 + B 3 5 + B 3 4 + A 1 3 + A 3 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Of specific interest is interviewee AL1 from the control group. She entered into Math I with a positive attitude, except where fractions were concerned. However, compared to all the other interviewees, she was the one participant who had negative change in more areas. She entered into this course with generally positive thoughts about mathematics and that did not change. However, her confidence in her ability to do mathematics and teach mathematics suffered greatly. She attributed this decline in confidence to poor performance during the semester. On a more positive note, her comfort level with fractions did improve. Her responses from the final interview contradict what she wrote in her journal. The interview was completed after the journal, and had more negative responses regarding her confidence in knowing and teaching fractions. In her final interview she said, “Like before this class I felt fine about them. I mean it is something I always need to review. Just like I said, in this class it kind of hurt my confidence in a lot of areas, including fractions (May 2006).”

There were two categories where all of interviewees from the experimental group showed positive gains in their beliefs about mathematics. The first is confidence in doing mathematics. In comparison, three of the six participants in the control group improved and one did not improve. AL1 was the one to suffer the only setback in this category. Closely tied to this area is fraction comfort level. Again, the entire experimental group improved, whereas only three of the control group improved and one declined. AL2 said that he was comfortable with fractions, but “I may be a little bit more confused with them since I’ve been in this class (May 2006).” When I asked why he was more confused, he decided that a better description was “overwhelmed” but in the end he did not think that fractions were more difficult. During this part of the interview he mentioned that the use
of manipulatives was a confusing part of this process. In the lecture sections, manipulatives are not used in the teaching process, but they are introduced as resources. Recall that AL2 had experience teaching fractions to students preparing for the GED and he mentioned at the beginning of the semester that fractions were a big part of that preparation. AI2 had a positive change in his opinion of fractions from the beginning of the semester. He started the semester with a complete dislike of fractions. At the end of the semester, he believed he will be fine teaching fractions and he does not dread them as he once did. He attributes this turn around to the way the unit was taught in his class. When asked to be specific, he said,

Just being able to work with your own peers and not have to sit there and go through the whole lecture thing. That seemed to help me out but it was also really hard to sit there and sort of have to teach yourself, on say, for instance why we invert and multiply. I think it’s a little bit easier to work in groups and your attention goes more onto your peers than to a teacher. That just changed how I felt and I learned a little bit more. (May 2006)

His most profound statement about how the fraction unit changed his outlook on fractions was written in his journal.

After studying and spending time on the fractions unit, I feel I will be alright teaching children why and how to solve fractions. I know I will make mistakes at first, but the more time I spend on fractions, it will only make me stronger and able to explain everything in a clear and easy to understand manner. I plan on working with fractions more often than ever before, so I will not make my children in my class confused and hate fractions as much as I did when I was there age. (May 2006)

BI2, an interviewee from the other experimental section who did not show the same amount of positive change, said that

Now, after this class, I realize that fractions are more than just a number. I have more trouble now understanding just exactly what they mean. In a way they are more complex and complicated. (May 2006)
A feeling that fractions were much more complex than just a set of numbers with computation rules was a theme that also emerged with these participants from the experimental group.

At the beginning of the semester, I asked participants to reflect on the type of mathematics instruction they remember from their k-12 education. Overwhelmingly, if students remembered hands-on work, it was in the lower grades. Middle school and high school math classes were remembered to be traditional in that class would start as a lecture and end with working problems alone or with their peers. I then asked interviewees to explain how mathematics instruction would be organized in their ideal classroom setting. The responses from the beginning of the semester were different between groups. To begin with, the experimental group all said that a mix of lecture and group work would work best. Only two participants in the control group thought that a mixture of lecture and group work would be best. Three thought that all lecture would be best and one thought that student-based instruction would be best. A problem that was encountered in this questioning was that the participants’ lack of teaching experience as well as their lack of education courses thus far in the program hampered their communication of their idea of ideal mathematics instruction. At the beginning of the semester most students were not aware of inquiry-based learning. However, they often referred to group work or work with manipulatives when they explained a departure from a lecture-based learning environment. This is why a “5” in this category was considered to be student-centered learning as opposed to inquiry-based learning.

From the control groups, AL2 said that he thought it would be better for his students to have more of a hands-on experience in math. He went on to say that,
I always learned better if I could sit there and work the problems out in the class and kind of know what I was doing and then go home and work on homework — and that’s what I want to try to do when I start teaching is do a lot of group work, work with groups and solve like real life problems, relate the subject to the real life problems and hopefully get students interested. (January 2006)

AL1 also wanted to incorporate group work but did not sound as sure. She said,

I definitely want to have a mix because I believe in group work, but also, through math, I’ve learned so much through lecture. So I want to have that too – seeing it taught on the board. I feel like when the teacher explains it to you like that, you have to see her work it out. (January 2006)

Neither participant elaborated on what she meant by group work. At the end of the semester, AL1’s attitude had changed a little. She said the class had been difficult for her and that at times she wondered how she was going to be a teacher. She went on to say that this was because “I felt like I knew it and then when it came test time or something I wouldn’t do well.” When asked about how she would teach mathematics now, she said she would like to use group work because students might not understand a concept from her but might understand it from a fellow student. She still wanted to show them how to do things on the board and then let them work together on problems. At the end of the semester, AL2 said that his ideal mathematics classroom would incorporate lecture followed by group work. When asked about the group work, he said he wanted to include real life problems. He said the problems would be something that would incorporate the entire section. BL1 had taken some educational course work prior to this class and said,

We watched a video in my Educational Psychology class and it really stuck with me because it was the constructivist idea. And the kids already knew basic math like single digit addition. And then when they put together the two digits, the teacher didn’t say a word and let the kids figure out how to do it. And the way they put it together in their head made a lot more sense than I think forcing them and teaching them how to work it out your way. So I would like to do where I teach them the basics, then give them a problem and let them work their own way through it. Kind of like scaffold them… (January 2006)
BL1’s conception of how she would teach mathematics changed in that she believed lecture might have an important place at times. This description of how she would like to teach mathematics was less student-centered than it was at the beginning of the semester. At the beginning of the semester, BL2 said that “Manipulatives would be a major, a major, major thing because I have seen children learn so much better with manipulatives and colors and stuff like that. And group work. (January 2006)” At the end of the semester, her beliefs about teaching had not changed and manipulatives were still really important to BL2. Since BL3 was an exceptional education major, he was unsure how he might be able to work with students because he felt there were too many variables to be able to talk about instruction. By the end of the semester, BL3 had decided he would like to be in a resource setting and that when helping children learn mathematics he did not want to give too many strategies because he thought that might be confusing for his students. However, he did feel like having learned different ways to look at mathematics would help him help students who struggle with learning. BL4 wanted to use lots of hands-on materials when teaching and that had not changed at the end of the semester.

At the beginning of the semester AI1 said she wanted to implement group work and fun hands-on activities. She said, “I would most definitely try to explain all I could by using objects rather than a chalk board and a piece of chalk (January 2006).” At the end of the semester she still has the same idea of how to teach mathematics but went on to say at first there will be no lecture, and then said,

I don’t like – I mean you have to have some sort of lecture but not just sitting in front of a podium and speaking. Not at all. Lots of group activities. Get the students’ opinions. Let them choose some assignments in a way I guess you can say. Or just have a lot of the students input. And of course using – drawing pictures now. I’ve learned, has helped me so much. It’s helped me understand more so obviously it helps kids understand more. And using
When AI2 was asked at the beginning of the semester how he would like to teach mathematics, he said, “Just different ways. Students – like some kids learn by more hands-on like games.” He went on to say, “…I might try lecture and more just other different ways, I guess (January 2006).” His view on how he might teach mathematics did not change during the semester. At the beginning of the semester, AI3 described her ideal teaching style in the following quote.

I want to be able to find several different ways or strategies that my students can learn, you know, and find one – I mean, obviously, math is all these strategies where you, depending on what you are doing in math, you learn different ways to solve problems or whatever, but I want to – I just want to be able to make sure that my students understand how they’re getting these answers, and how – I don’t know – just how – I just want (them) to be able to understand, I guess the concept behind and all that stuff. (January 2006)

After a semester of this mathematics class, AI3’s expectation for teaching mathematics changed only in that she was surer about what she was saying. At the end of the semester she said she wanted to make sure that she puts “the concept behind my teaching mathematics into it. And I don’t want to give my students the pattern and say, ‘Solve it.’ I want to say, ‘You know, this is how you get the answer and this is why. Why it works. (May 2006)’”

Initially, BI1 said “We would do a little introduction to what we were going to be doing… It would be a little bit of lecture and then a question and answer session with the kids, seeing if they’ve caught on to what I said in the lecture. And then get with a partner and work together like that (January 2006).” This changed during the semester to “It’s going to be – I think it would be most beneficial for it to be hands-on and not so much lecture. Because you have to have lectures, but the hands-on is what really helps people
to see what’s going on (May 2006).” BI2 was not really sure about how she will teach math at the beginning of the semester. At the end of the semester she says she will teach by taking more of an angle by focusing on why we do something so that students will understand more. She then says, “Probably using like materials instead of just trying to show it on the board or something. Maybe some kind of blocks or something like that” (May 2006).”

Overall, the quantitative results for this portion of my study did not reveal that method was a significant factor in improving beliefs about mathematics or the teaching of mathematics. However, there was indication that an inquiry-based approach did not harm beliefs or attitudes even if it was implemented in the middle of a semester. The interviews and other self-reported data indicated that inquiry might have a more positive effect on general thought regarding mathematics, confidence in doing mathematics, and participant’s comfort level with fractions.

Summary

In this chapter I have examined preservice teacher knowledge as it relates to fractions and how the changes in this knowledge change in an inquiry-based approach versus a lecture-based approach. I first reported on 94 preservice teachers’ conceptual understanding of fractions based on an assessment given at the beginning of the fall 2005 semester. I corroborated this with interviewees conducted during this same time frame. The results point toward weak conceptual understanding of fractions and their associated algorithms. I then reported results from the data I collected in the spring of 2006 on whether an inquiry-based approach had was more advantageous than a more traditional lecture-based approach. The results indicate that an inquiry-based approach does have its
advantages over a lecture-approach. In addition, I found evidence that the inquiry-based approach did not hurt the acquisition of skills and that retention might increase with an inquiry-based approach.

I now turn to Chapter 5 where I draw conclusions based on the results of the analysis of the data collected during this study and I explore some of the issues that analysis of the data raised.
CHAPTER FIVE
CONCLUSIONS

Introduction

This chapter provides a discussion of the conclusions, limitations, and implications of this study. The chapter begins with a discussion of the results for each of the questions this research study set out to answer. The next section provides an evaluation of this study and addresses the limitations that existed during the research and offers possible adaptations for replicate studies. The last section addresses the implications of the study for the preservice mathematics community and what future studies might follow as a result of this research.

Conclusions

Question 1

The first part of this study set out to determine what preservice teachers know about fractions before completing their required mathematics content courses. In my experience, I observed that some of the students in these courses possessed weak procedural knowledge and that many had weak conceptual knowledge. However, I was surprised by just how weak many of these students are in numerous areas. The participants in this study were not just weak in conceptual knowledge but also in procedural knowledge that they should have mastered before entering high school. This is problematic because of their intended careers and their general attitudes about teaching mathematics. Many of these students enter into this course sequence at this university with the belief that they will not teach much mathematics at the elementary school level. When I share student
work from a local elementary school, these preservice teachers are often shocked at the high level at which local elementary school children are working. Most often this shock is related to how young children are exposed to topics in mathematics that they thought would be introduced in a higher grade level. In addition, they are at times shocked at the depth of understanding elementary children are expected to possess.

Looking at the results of the pretest from the fall semester, from the cognitive domain of knowing, there were only two questions on which more than 75% of the participants answered correctly. The concepts that participants had the most difficulty with were ordering of decimals and fractions and equivalence. These are basic concepts that are a critical foundation in understanding rational numbers. On 3 of the 5 questions using routine procedures, fewer than 53% participants answered correctly. These were problems dealing with basic operations. One of the problems was a division of fractions problem and only 48% answered correctly. On the 5 problems that required more thought, the highest percentage of participants that answered any one problem was 76%. These results are most disturbing because they were from the 1999 TIMSS test created to assess 8th graders.

On Part 2 of this test, the participants’ lack of conceptual knowledge was highlighted. Students were asked to solve two application problems that were designed for the 8th grade in two ways – algorithmically and pictorially. For the addition problem, only 37% were able to set the problem up and reach a correct answer algorithmically and only two percent were successful at solving the problem by drawing a picture and explaining the process. Another 19% drew an initial picture and a picture for the answer, but did not explain the process. For the division problem only 20% answered using an
algorithm and 14% showed they understood conceptually how a picture could be used to solve it. These participants’ inability to set up and solve a simple application problem using a picture indicated a lack of conceptual understanding of fractions and their operations. These results were confirmed during the interview process.

The interviewees showed a lack of conceptual understanding with both addition and division of fractions. In general, the participants were unable to verbalize why we get common denominators to add fractions when we use the standard algorithm. One of the problems I saw was that when the participants were telling me what a fraction was, they did not place importance on the size of pieces a whole was broken into. For example, the fraction $\frac{2}{3}$ was most often described as 2 parts of 3 without placing emphasis on the “whole” being divided into three equal pieces. When drawing a picture to represent an addition problem, the interviewees did not encounter difficulty when the denominators were already the same. However, when the problem had unlike denominators, interviewees could not solve using pictures, unless they found a common denominator first. They were unable to see how to take a picture of the fraction $\frac{2}{3}$ and change the picture into an equivalent fraction such as $\frac{4}{6}$. From Part 1 of the assessment, we saw that equivalence was one of the areas in which the participants encountered difficulty. These participants were also unable to explain why the standard algorithm for adding fractions worked.

The interviewees showed even less understanding of fraction division. Most of the interviewees possessed the procedural knowledge necessary to solve a simple division problem. However, though 14% of all participants were able to use a picture to solve a
division problem in context, none of those interviewed were able to use pictures to solve the division problem, including one student who had been successful on the pretest the first day of class. For her, having the problem in context was helpful. During the interviews, I discovered that many of these participants did not possess conceptual understanding of whole number division, a logical prerequisite to understanding division of fractions. It was not surprising that none of the participants could explain why we invert and multiply when dividing fractions.

I also looked at 13 students who had previously taken the course and earned a D or F. The average score for the 13 repeaters was 64% versus 73% for the non-repeaters. This is discouraging since the exposure they had in a previous semester should have given the repeaters an advantage over the non-repeaters; however, they were likely to be weaker students to begin with. Nevertheless, this is cause for concern since there is a push for highly qualified teachers across the country and one of the necessary prerequisites is content knowledge. There are students at Southeastern University that take Math I and Math II numerous times before passing. This also raises good questions for further research. Why are so many students under-prepared for these courses? Will a change in instruction help them succeed even if they are under-prepared? Are the prerequisites currently in place appropriate? If not, what prerequisites will help under prepared students the most?

It is important to note that the average ACT mathematics score was 20.06 for participants in this study and that the national average ACT mathematics score in 2005 was 20.7 (ACT, 2006). While the participants in this study had a lower average score on
the mathematics portion of the ACT, it is reasonable to believe that the results of this study could be expected at other universities as well.

Question 2

The second question that I sought to answer was if preservice teachers who have completed an inquiry-based course possess better conceptual understanding of fractions and the standard algorithms associated with addition and division than preservice teachers from a lecture-based course. The results indicate that inquiry does have an impact on conceptual understanding.

The multivariate analysis of covariance (MANCOVA) for the full sample showed that there was a significant difference between the control group and the experimental group. Once I determined that there was a difference between methods, I examined the individual analyses of variance (ANOVA). From this section of results, I could see that there was a difference in final exam scores and the difference scores for posttest-pretest. The adjusted means confirmed that the students in the inquiry classes had higher mean scores on these test measures. While all of the tests were tests of fraction knowledge, the final exam and Part 2 of the pre/post test were designed to test conceptual knowledge. The experimental group showed the significant gains in conceptual knowledge based on the post-pre means in comparison to the control group. I attribute this to the time students spent investigating problems in context to determine why the standard algorithms work.

A focus of fraction content in Math I is to get students to understand why common denominators are necessary to add fractions with the standard algorithm. On the final exam the participants were asked to explain this straightforwardly and 68% of the inquiry-based group answered correctly and 70% of the lecture-based group answered
correctly. Participants were also given \( \frac{2}{9} + \frac{5}{9} = \frac{7}{9} \) where the unit is fixed and asked to explain why the answer was not \( \frac{7}{18} \). Thirty-five percent of the inquiry-based group answered correctly compared with only 10% of the lecture-based group. While the success rate on each question was not the same, the inquiry-based group still had more people responding correctly on the second question. Since this question was posed differently than on prior tests and quizzes, there is an indication that more inquiry-based participants truly understood the reason for common denominators when using the standard algorithm.

Another exciting result to consider is that there was not a significant difference in scores with the quiz or test but there was a significant difference with final exam scores. Both the quiz and the test were given during and right at the conclusion of the fraction unit. However, the exam was given several weeks later. This is fascinating because it suggests that there is a knowledge retention factor at work when inquiry is used as the sole means of instruction. In addition, even though skills were not emphasized in the experimental sections, these students still performed as well as the control group on Part 1 of the pre/posttest, which is skill based. This too is an important result that I will discuss further when I address limitation and implications.

*Question 3*

For the last question of this study, I sought to find out if an inquiry-based approach improves the attitudes that preservice teachers have about mathematics and the teaching of mathematics. While there was overall positive change during the semester, the difference between the experimental and control group was not significant. There was
one disconcerting result that presented itself in the quantitative analysis of the Fennema-Sherman survey results. There was a three-way interaction between method and instructor over time. The interaction was present for a couple of reasons. The students in instructor A’s lecture-based class had a decline in beliefs from pre to post in her lecture section, whereas her inquiry section show improvement over time. Instructor B had improvement in both sections, but her inquiry-based section showed a sharper gain over time. These results were confusing and not what either instructor expected. After this interaction was discovered, the other instructor and I discussed this interaction but we had no verifiable explanation for this result. What we know to be true is that many preservice teachers enter Math I with poor attitudes regarding mathematics and a pessimistic attitude regarding this course sequence. Part of this attitude towards the class comes from rumors about how difficult the courses are and how many people have to retake the course. In addition, the standards for getting a “C” in the course are higher than in all other mathematics courses. We determined together that it was not necessarily surprising for a group’s attitudes to decline during a semester of this course. However, that does not explain why instructor A’s lecture-based section showed a decline while the participants in her inquiry-based section showed an improvement in attitudes relating to mathematics. While not verifiable with the data at hand, it is possible that instructor A was more predisposed to teaching in an inquiry-based setting and instructor B was comfortable in both settings.

The results for the survey of teaching beliefs found no significant differences for method. However, it is important to point out that there was an overall improvement in beliefs about teaching for the semester. This means that overall, students’ beliefs were
more in line with an inquiry-based approach to teaching at the end of the semester in comparison to the beginning of the semester. The results from both parts of the survey showed that even though inquiry was implemented for only a single unit, it did not have a negative effect on student attitudes or beliefs. A future study might look at the effects of using inquiry for an entire semester and look at attitudes about mathematics and beliefs about teaching mathematics.

Instructor Change

In an effort to strengthen this study, I chose to involve another instructor in this research. At Southeastern University, there was not an instructor with experience teaching in an inquiry-based or student-centered classroom, so I invited an instructor who was open to trying something new in the classroom. While she was glad to volunteer, she was still anxious since she had not taught or learned in a classroom environment that was centered on the student. She reported that after we piloted the lessons in the fall of 2005, her anxiety level was reduced. However, she was still nervous. Her biggest concern was that she would tell answers too readily when students asked questions of her without requiring them to think through things for themselves.

After being involved in this project for a year, she is excited about incorporating inquiry-based learning into Math 1 and Math 2 in future semesters. At the end of the data collection phase, she said,

I feel comfortable with using inquiry-based learning in my classroom now. I am disappointed that I am not teaching Math 1 this fall because I am excited about incorporating some of the inquiry based learning into my Math 1 classes.
Now that she had this experience in one class, I wanted to know how that might impact other classes as well and she said,

I do feel my teaching will change in my Math 1 classes and maybe in my Math 2 classes. I am not sure it will change in my College Algebra classes but I would like to try and find ways to incorporate some inquiry-based lessons in those classes too.

This study shows that even if an instructor has limited or no experience teaching using an inquiry-based approach she can still try with the right curriculum and support. In turn, this instructor serves as a good example that, with the willingness to try something new, minimal professional development, and support, instructors may be successful teaching with inquiry.

Limitations

While there were several interesting results and important implications from this research, there are several areas that should be expanded upon to gain further insight into preservice teacher learning.

With regard to the method that was utilized for this study, there are a couple of confounding variables. First students self-select themselves into a particular section based on time-of-day, the assigned instructor, or day of course. However, students were not aware of the study in advance so they could not intentionally self-select into the control or experimental group. The most important consideration is for the day sequence – Monday-Wednesday-Friday (MWF) or Tuesday-Thursday (TR). One instructor held classes on MWF for 55 minutes and the other on TR for 80 minutes. There are various schools of thought on why a student my select one over the other. However, some would
argue that a MWF section for inquiry is better because there is more exposure to mathematics and some would argue that TR section is better for inquiry because there is a longer period of uninterrupted time for the lessons. This would certainly lend itself to an interesting area for future research. Does an inquiry-based learning work better in a MWF or TR section?

Second, students could come to an instructor’s office for additional help outside of class. These hours were not tracked for either instructor. However, while there are always students who are more invested in doing well, there were not a disproportionate amount of students coming for help in any one class. In addition, the other instructor and I discussed that it was important to treat a student from an inquiry-based section in the same manner as we would treat them in class if they asked a question.

Another limitation relating to the method employed in this study was that while I observed every inquiry-based lesson the other instructor taught and used a fidelity checklist to ensure that she did what was expected during the lesson, there was no one to observe my classes to verify what I did in class. All that was available were the videotapes of the lessons that I viewed when analyzing my data. These videotaped observations did document that I followed the written lesson. However, there is the possibility that with my experience teaching in an inquiry-based environment, I may be more capable in utilizing these methods. Even if this was the case, there was not a significant difference between instructors and our inquiry-based classes performed equally well.

While the lessons were carefully written, piloted, and revised, this is an ongoing process. Although several people with varying levels of expertise had a hand in writing
and approving these lessons, there is always room for improvement. These lessons were written with a specific target audience in mind and therefore might not work as they are written at another university if the preservice teachers are not performing at the same level upon entry to this course sequence. Student and instructor feedback on the lessons was in general positive. Instructor observations during the pilot and student suggestions, both solicited and unsolicited, were used from the pilot to improve upon the lessons. However, even with these improvements, during the experiment, instructor reflections and observations I noted that there was still an issue with the lessons. In particular, in the division activity, students had difficulty with envisioning the process with the entire divisor.

Another limitation to this study is that inquiry was utilized only during the fraction unit itself. I believe it would be unrealistic to expect a significant difference in beliefs about mathematics or teaching mathematics with limited exposure to inquiry-based teaching and learning. With a full semester or more of exposure to an inquiry-based style of teaching, I would expect significant improvements in beliefs as they relate to the teaching of mathematics. With more resources and time, more inquiry-based units could be designed to allow for this increase in exposure.

Another limitation was the short amount of day-to-day class time to devote to inquiry-based learning. Lecture is an efficient method to cover a lot of material in a short period of time. Math I is full of content that can be difficult enough to complete in a semester using lecture. To get the full effects of inquiry-based learning, the amount of material would need to be reduced. While this is a drawback to many educators, I saw evidence this semester that even though we did not expressly cover an idea in the
experimental section, the students were able to draw on other knowledge to complete the
same problems with the same success rate as the control group. For example, the concept
of finding a fraction between two fractions is a concept that was expressly covered in
class for the control group. However, the experimental classes did not discuss this topic
and only one related homework problem was assigned. In the control group, students talk
about the density property and are shown how to find a fraction between two fractions
(by adding the numerators and denominators). From the quiz, participants were asked to
find one fraction between $\frac{1}{7}$ and $\frac{1}{8}$. Thirty-six percent of the lecture-based group missed
this question compared to only 18% of the inquiry-based group. All the correct responses
fell into two categories. They either solved the problem using a theorem
\[
\left( \frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d} \right)
given in the lecture-based section or they relied on other fraction
knowledge, such as equivalent fractions. Twenty-one percent of the control group used
the theorem and only 3% of the inquiry-based used this method, whereas 76% of the
experimental group answered using conceptual knowledge compared with only 22% of
the control group. These results show that even though the experimental group did not
receive explicit instruction on this topic, they were able to use other fraction knowledge
to answer the question. A couple of weeks later on the unit test, students were asked to
find two fractions between $\frac{1}{8}$ and $\frac{1}{9}$. Fifty-eight percent of the experimental group
answered correctly compared with only 39% of the control group. There was a dip in the
correct responses between the two groups, likely because of the request for two answers.
Several in the control group were unsure how to use the theorem to help find a second
fraction. Clearly the lack of explicit instruction did not hinder participants in the experimental sections.

Implications

This study evolved from observations that I, as well as fellow mathematics educators, made while teaching the content courses at Southeastern University. The implications of this study fall into two major categories: preservice teacher knowledge and curriculum and instruction design for preservice teacher mathematics courses.

Preservice Teacher Knowledge

As I discussed in Chapter 4, most of the preservice teachers in this study reported having traditional mathematics experiences throughout their K-12 education. As a result, their focus when learning mathematics had been to memorize how to do something, not to understand why things work the way they do. There is a lack of conceptual understanding as a byproduct of learning this way. While memorization may have its place in mathematics, it is imperative for preservice teachers to possess the necessary procedural and conceptual content knowledge they will be teaching. For example, knowing how to divide fractions by memorizing that we invert and multiply when dividing fractions is sufficient to compute the answer to \( \frac{2}{3} \div \frac{1}{2} \); however, in a real world problem, conceptual knowledge is sometimes necessary to be able recognize how to set up a problem and then solve it. This was evidenced in the pretest during semester one. There were students who exhibited sound procedural knowledge when dividing fractions. Nevertheless, when given an application problem involving division of fractions, they did not understand enough to recognize that it was a division problem. Liping Ma (1999) observed this lack of conceptual knowledge when she asked teachers to write an
application problem that involved division of fractions. As I reported in Chapter 2, most of the teachers in her study were unable to complete this task correctly. If preservice teachers possess both procedural and conceptual knowledge they will be better able to help their students understand more and memorize less.

The participants’ difficulty in these areas indicates poor conceptual knowledge that should be addressed before they enter the teaching profession. What seemed to be clear from the results was that we, as educators, must first address conceptual understanding of whole numbers. Then we must address conceptual understanding of fractions before the main focus shifts to conceptual understanding of fraction operations.

Curriculum and Instruction Design

My study, as well as other studies (Cobb, et al., 1991; Villasenor & Kepner, 1993; Carpenter, et al., 1989; Wearne and Hiebert, 1989), indicates that learning in an inquiry-based program promotes conceptual understanding without harming skills. Inquiry-based leaning allows for students to focus on the depth of their learning that allows for transference of ideas to new situations. Therefore, choosing or developing an inquiry-based curriculum that supports preservice teacher learning is important. Moving in this direction addresses several important issues that preservice teachers will face. One is that the right curriculum will allow these preservice teachers to learn in an environment where inquiry is not only encouraged but expected. With the indications that this method of learning can promote conceptual understanding without the loss of procedural knowledge, preservice teachers could be at an advantage. Since most of them are products of traditional lecture-based mathematics experiences, they will most likely fall back on lecture when they enter the classroom. However, with full exposure to an
inquiry-based learning environment for two semesters, they will have exposure as a learner to a method of teaching that would help them with pedagogical issues they will face in the classroom.

Teaching in a student-centered environment can pose difficulties if instructors are not trained to teach using this approach. The instructor that agreed to take part in this study did not have a great deal experience teaching using a student-centered approach and she had no experience teaching using an inquiry-based approach. Her willingness to be part this study and her openness to change helped make this study a success. Not all instructors at this level are willing to change and sometimes the structures are not in place to support instructional changes of this nature. For instruction to change from lecture-based to inquiry-based there is a need for training and ongoing professional development for those professionals who wish to move away from more traditional means of teaching. The instructor that assisted in this study received minimal training but was a willing participant and motivated to try something different. As she reports, the success she had with teaching this way will affect the way she will teach these courses in the future.

Future Directions

This research study looked at only a very small portion of preservice teacher learning in the mathematics classroom. While this project is drawing to a close for the purpose of this paper, the research will continue in an effort to improve the existing inquiry-based lessons, and expand them to encompass more content. Within the upcoming year, Southeastern University is beginning a project to redevelop Math 1 and Math 2. This redevelopment is to realign the current curriculum with the content preservice teachers need when they enter the classroom. This study will play a role in
informing that redevelopment of both the curriculum and methodology that will take
place during future semesters. In order for this to happen, more research with more
lessons is needed. In addition, in order to generalize any findings from this study, it
should be replicated at other universities. In addition, this study should continue at
Southeastern University when more lessons are developed to ascertain whether inquiry-
based classroom are beneficial with other content.

Another consideration for future research concerns knowledge retention. One of
potential drawbacks about lecture-based learning is that it is not focused on conceptual
understanding is that students memorize what they need to know for the test only to
forget it soon after they take the test. This study showed that the lecture-based and
inquiry-based groups performed the same on the assessments given during or
immediately following the unit; however, the inquiry-based group performed better on
the final exam and the posttest. Will the experimental group retain fraction knowledge
longer than those who learning in a lecture-based environment? The indications from this
study suggest the answer is “yes,” though future research can be used to confirm this.
How long these gains will last is another issue for future study.

Conclusions

In order to answer the research questions that I set out to address, it was necessary
to examine what preservice teachers knew about fractions. The results from this portion
of my research are alarming. Many of the participants had trouble working problems that
were developed for 8th grade students and the results indicated that many of these
preservice teachers have gaps in their fraction knowledge. I then set out to determine if
the preservice teachers who have completed an inquiry-based course possess better
conceptual understanding of fractions than preservice teachers who were exposed only to lecture. Once I established that there was a difference in the groups where method was concerned, I had to identify if this difference was in all of the test measures or only in a select group of the test measures. What I found was there was a difference when inquiry was used and that these students did possess more conceptual knowledge at the end of the semester. This is an exciting result and it sets the stage for further studies with the use of inquiry to increase the conceptual knowledge of preservice teachers. These results were cause for further excitement because there is also an indication that knowledge retention might be better with an inquiry-based approach.
APPENDICES
APPENDIX A

Conceptual Understanding of Fractions

Objectives:

✓ Focus on number sense that enhances the conceptual understanding of fractions
  o Refocus from the fraction symbol to the relative amount that the symbol represents
  o Understand equivalent fractions and writing fractions in lowest terms
  o Ordering of fractions / Estimation

Task One (Relative Amounts):
Students will be given a handout with several pictures that represent the number 3 and several pictures that represent different fractions that at first glance look to represent different amounts, but instead represent the same relative amounts.

§ After students represent how much each picture represents, discussion will center on how the pictures are different, how they are the same and that \( \frac{1}{4} \) refers to the relative amount shaded. Discussion will also compare how “3” always represents 3 items but that a fraction like \( \frac{1}{4} \) can represent many different quantities and why this idea of relative amount is so difficult to grasp.

§ To further exemplify this concept, the instructor will show groups of three items and ask what numeral represents that number of items.
  o Three “hugs”
  o Three Legos
  o Three quarters, etc.

§ The instructor will also show real life examples of fractions. For example, the instructor can choose (based on items they have on hand) to show the fraction \( \frac{3}{4} \) (or any other fraction) by showing:
  o \( \frac{3}{4} \) of a cup
  o \( \frac{3}{4} \) of a pitcher
  o \( \frac{3}{4} \) a dollar (shown many ways), etc.

§ In this discussion, it is expected that some students will recognize equivalent fractions which will lead into next task. Also, based on earlier tasks from the previous day, students might recognize that in one of the examples a unit might consist of three circles.

Task Two (Equivalent Fractions / Lowest Terms):
Students will be given a set of Cuisenaire Rods to use in groups and a set of paper “rods” that they can use at home. It is expected that most students have not worked with Cuisenaire Rods but there will not be explicit instructions on how to use the rods. As students progress through this task, they will learn to use the rods to explore the idea of a “linear region model” to identify equivalent fractions and how to write fractions in lowest
terms. The following are questions that are on the student handouts (This lesson was adapted from a lesson retrieved from the Illuminations web page at http://illuminations.nctm.org/LessonDetail.aspx?id=U152). A discussion of these concepts will follow the activity. This discussion and the post assignment will lead to how a student can tell if fractions are equivalent and how a student can tell if fractions are in lowest terms.

During the discussion, the instructor will ask students what equivalent fractions are and how they can tell when they have equivalent fractions. Their answers should relate to the specific activity and to what they know about fractions in general. The instructor should also ask the class how they can tell when fractions are in simplest form and tie this concept into equivalent fractions. The instructor is also asked to find out if the class can think of why using the terminology “reducing” fractions can be misleading to students.

**Student Handout:**

The items are below are some of the problems selected from the student handouts.

1. If white = 1, what value would you assign to all the other rods? Complete the table (included on student handouts).
2. If pink (red) = 1, what value would you assign to all the other rods?
3. If dark green = 1, what value would you assign to all the other rods?
4. If black = 1, what value would you assign to all the other rods?
5. If orange = 1, what value would you assign to all the other rods?
6. You can create pieces by combining two colors together. Create an orange/red by placing one of each together and let it equal 1. Now find the value of the other rods.
7. Let’s create another color – dark green/black and let it equal one. Now find the value of the other rods.
8. In each of the problems 1-7, what was the value of the dark green rod? How did this dark green rod have a different value in each problem?

Now let’s explore some specific fraction relationships.

9. What colors can be lined up end-to-end to create the same length as the brown rod? You can not mix colors, the rods must be the same color. For example, eight white rods can be lined up to create the same length as one brown rod. So what other rods can be lined up to create the same length as the brown rod? Sketch the representation.
   a. Using your sketch and the idea that the brown is the unit (brown=1), assign values to each of the rods in your sketch.
   b. Using the values you assigned in part a, name as many fraction relationships as possible.
   c. What do you call the fraction relationships you listed above?
   d. What does the group with the smallest number of rods represent?
   e. Identify the fraction that is in lowest terms from each of the equivalent groups mentioned above.
10. Now create a new “color” rod. If you combine an orange and yellow rod you get the color orange/yellow. What colors can be lined up end-to-end to create the same length as the orange/yellow rod? You can not mix colors, the rods must be the same color. Sketch the representation.
   a. Name as many fraction relationships as possible.
   b. Which fractions are in lowest terms? How can you tell by looking at your sketch?
   c. How can you tell when your fractions are in lowest terms without looking at your sketch?
   d. How can you tell when a fraction in simplified form?

Task Three (Ordering Fractions/Estimation):
In this task students will be asked to use reasoning to order fractions and then make generalizations on how to order fractions. Students will be given the following questions to aid in this process. Groups will present their generalizations. Throughout this activity and throughout the discussion, students should be encouraged to think about fractions so that they can reason through each of the activities. After students have finished this activity, the instructor will ask students to put solutions to problems 1-7 on the board. Students will explain their thinking on each of the problems for the class. The instructor will ask if there we different ways to do each of these.

Student Handout:
1. A unit fraction is a fraction whose numerator is one. Given the following three unit fractions $\frac{1}{7}$, $\frac{1}{3}$, and $\frac{1}{5}$, put them in order from smallest to largest and explain how you know this is the correct order.

2. Draw a picture of the following two fractions $\frac{4}{5}$ and $\frac{3}{5}$ to determine which one is smaller
   a. Use words to describe this relationship. Now use symbols to describe this relationship.
   b. Explain how you used your picture to help find the order of these two fractions.
   c. Explain how you could order these two fractions without the use of your picture.

3. Now draw a picture of the following two fractions $\frac{4}{5}$ and $\frac{4}{7}$. Determine which one is smaller.
   a. Use words to describe this relationship. Now use symbols to describe this relationship.
   b. How did you use your picture to order these two fractions? Did you encounter any difficulty? Was your picture helpful?
   c. How could you order these two fractions without a picture?
4. Using the same method you used in the previous two problems, can you order \( \frac{7}{10} \) and \( \frac{9}{15} \)? Can you think of a better way to order these fractions? If so, what is it?

5. Mentally determine which fraction is larger, \( \frac{4}{7} \) or \( \frac{1}{2} \) without rewriting the fractions in other terms. Is it easy to compare these two fractions? If so, why?

6. Using what you know from ordering the fractions in number 5, which fraction is smaller, \( \frac{7}{13} \) or \( \frac{5}{11} \)? Explain your reasoning.

7. Extending the concept in 5 and 6, which fraction is greater, \( \frac{3}{8} \) or \( \frac{4}{10} \)? Explain your reasoning. Are you comparing these fractions to a benchmark of \( \frac{1}{2} \)? Why or why not? If you are not comparing these fractions to \( \frac{1}{2} \), what did you compare the fractions to and explain why you chose that benchmark.

8. In problems 1-7, you ordered fractions using different methods. Can you generalize these processes so that they can be used in other examples that are similar?
   a. To order fractions that have the same denominators with unlike numerators you ….
   b. To order fractions that have the same numerators with unlike denominators you …
   c. To order fractions using \( \frac{1}{2} \) (or another numeral) as a benchmark you…
Conceptual Understanding of Fractions
Homework Assignment

Students will be given the following on a sheet to complete for homework. The assignment will be turned in at the next class.

1. Solve this problem using two different methods and explain your reasoning. There are 14 sandwiches to be shared equally among 8 people. How much will each person get? (Schifter, et al., 1999b, pg. 57)

2. Jorge has two pizzas, one pepperoni and one cheese. Each pizza is the same size, and each is cut into 8 equal slices. Jorge eats 2 slices of the pepperoni pizza and 1 slice of the cheese pizza. (Schifter et al., 1999b, pg. 57)

   Use these facts to write three different application problems about Jorge’s pizza eating, one for each of the following answers.

   a. \( \frac{3}{4} \)
   b. \( \frac{3}{8} \)
   c. \( \frac{3}{16} \)
   d. In the list of facts it states that each pizza is the same size. Is this necessary? Why or why not?

3. Order the following fractions from smallest to largest using methods from class. Be sure to explain your reasoning for each problem so that it is clear how you placed the fractions in order.

   a. \( \frac{5}{9} \) and \( \frac{5}{7} \)
   b. \( \frac{8}{9} \) and \( \frac{7}{9} \)
   c. \( \frac{7}{12} \) and \( \frac{5}{12} \)
   d. \( \frac{10}{11} \) and \( \frac{11}{12} \)
   e. \( \frac{2}{5} \) and \( \frac{3}{7} \)
   f. \( \frac{8}{11} \) and \( \frac{9}{12} \)
   g. \( \frac{6}{11}, \frac{2}{5} \) and \( \frac{3}{7} \)

4. Find two fractions between \( \frac{1}{4} \) and \( \frac{1}{5} \) without changing your numbers to decimals. Make sure that your reasoning process is clear.
APPENDIX B
Addition and Subtraction of Fractions

Objective

- Use contextual problems to develop an understanding of addition and subtraction of fractions using pictorial representations.
- Use conceptual knowledge and contextual problems to see that sometimes common denominators are not needed to add fractions when the denominators are related.
- Use pictorial representations to build towards an understanding of why it is useful to have common denominators when using the standard algorithm for addition and subtraction of fractions.

Task 1: Addition and Subtraction of fractions with like (or related) denominators using models

Small Groups Exercise:
In this exercise, students will use only drawings (and manipulatives if they choose) to solve each of the following problems. If students choose to use manipulatives then they should draw the “process” so that they can share how they solved the problems.

1. You and your roommates go out for pizza. You order two large pizzas and there is \( \frac{3}{8} \) of one pizza left and \( \frac{2}{8} \) of the second pizza left. You all want to take the leftover pizza home so you choose to combine it into a single container. How much pizza are you taking home?

2. You and your roommates head to another restaurant for dessert. The pies at this restaurant come highly recommended and you cannot decide on which kind you want to try so you order two whole pies. As much as you all would like to eat all of the pies, you are unable to and place all the leftover pie into one box. You end up taking home \( \frac{1}{4} \) of one pie and \( \frac{3}{8} \) of the second one. How much pie are you taking home?

3. You are trying to be more diligent about drinking enough water during the day. Based on your body weight it is recommended that you drink \( 8 \frac{2}{3} \) glasses of water. If you have already had \( 3 \frac{1}{3} \) glasses how many do you have left to drink?

4. How are these problems the same? How are they different? Can you solve them the same way? Why or why not? (Would expect to get that two are addition one is subtraction, there are some fractions and mixed numbers, two have common denominators (cd) and one problem there is not a cd. They can solve them the
same way because they are drawing pictures and the second problem has related
denominators)

Students will be asked to put solutions for each of these problems on the board which
will be discussed. It is expected that students will notice and bring up the idea of common
denominators. It is hoped that someone will notice that you can solve the second problem
without common denominators because the denominators are related (one is a multiple of
the other). If not, through discussion of these three problems and their solutions, the
instructor will ask the students questions to lead to the idea that they could have solved
the second problem without common denominators.

Task Two:
Small Groups Exercise:
In this exercise, students will be asked to solve these problems two ways. The first
way is to think about how the fractions are related and try to come up with an answer
mentally. In the second way, the students will once again use only drawings but this time
they will be encouraged to use manipulatives (fraction tiles, Cuisenaire rods, or fraction
circles) to solve each of the following problems. These problems are designed to have
“related” denominators to make using models slightly easier to use in an effort to build
towards using common denominators. Students will be asked to pay close attention to
problems they encounter when solving each of these problems, in modeling the problem
with drawings and with using the manipulatives.

1. I am baking a special loaf of bread for a friend. The recipe calls for \( \frac{1}{2} \) cup of
white flour and \( \frac{1}{4} \) cup of whole wheat flour. What is the total amount of flour that
this recipe calls for?
2. My dog is \( 3\frac{3}{4} \) years old and my cat is \( 2\frac{3}{8} \) years older than my dog. How old is
my cat?
3. We recently repainted the living areas in our home. We overestimated the paint
we needed so we had \( 2\frac{1}{3} \) gallons of paint left over. If we use another \( 1\frac{1}{6} \) gallons to
paint the master bathroom, how much paint will we have left for touchups?

Students will once again put their solutions on the board to discuss their thinking with
regard to “related” denominators and how they utilized models and manipulatives (the
overhead will be available to show the use of manipulatives to the class). They should
notice that they did not need to find a common denominator to solve these problems since
they were not using the standard algorithm and each problem had related denominators
(where one denominator is the lcd). In the discussion, there will hopefully be students
who notice this concept of related denominators and how to tell when this relationship exists.

**Task Three:**

**Quick check:** (With a good foundation with fraction concepts, students should be able to add or subtract like fractions immediately). Compute the following quantities without modeling. Explain the process(es) you used. Could this process(es) be generalized to be used in certain situations? If so, what are those situations?

1. \[\frac{2}{4} + \frac{3}{4}\]
2. \[\frac{3}{12} + \frac{7}{12} + \frac{3}{12}\]
3. \[\frac{5}{8} - \frac{1}{8}\]
4. \[\frac{8}{15} - \frac{4}{15}\]
5. \[\frac{2}{3} + \frac{1}{6}\]

6. Explain the process(es) you used.

Discussion for 6 and 7 will highlight that when there is a common denominator, you just add numerators and leave the denominator. Last one can not be done this way unless you draw picture to get answer so need another way.

7. Could this process(es) be generalized to be used in certain situations? If so, what are those situations?

**Task Four:** (Using pictorial models to build towards the algorithm for unlike denominators)

Students will be asked to solve the following problem in small groups

1. Consider \[\frac{5}{8} + \frac{2}{4}\]. Take a moment and use manipulatives of your choice (fraction tiles, fraction circles, Cuisenaire rods, or fraction strips) to get the result.

   a. Did you change this problem into one that is just like the easy ones from the quick check where the parts (denominators) are the same?

   b. If so, how did you do this? If not, try to “convert” the problem \[\frac{5}{8} + \frac{2}{4}\] into one that has like denominators.

   c. Use your models to show the original problem \[\frac{5}{8} + \frac{2}{4}\], the “converted” one, and your solution, on your paper.

2. For the following problems, use your manipulatives to “convert” the original problem to a problem that has common denominators. In each problem, you
should use models to show the original problem, the converted one, and the solution on your paper.

a. \( \frac{5}{6} + \frac{2}{3} \)

b. \( \frac{7}{4} - \frac{1}{2} \)

c. \( \frac{2}{3} + \frac{1}{4} \)

d. \( \frac{5}{6} + \frac{3}{8} \)

e. \( 3\frac{3}{4} + 2\frac{3}{5} \)

f. \( \frac{7}{8} - \frac{1}{3} \)

3. Look at the process you used on each of the problems above. Did you convert each of the problems the same way? Why or why not? Generalize in writing how you converted each problem to a problem with like denominators. If you used more than one process, explain both (cd vs lcd). Was it necessary to convert all of these problems to problems with like denominators? Why or why not?

Students will share some of these solutions on the board. This will be followed by a discussion of the generalizations and how they relate to finding common denominators to add or subtract fractions using the standard algorithm and why it is necessary when using the standard algorithm.

**Task Five:**
Students will solve the following problem which will be followed by discussion.

1. Logan loves to play baseball. In his game Tuesday night, he made 2 hits out of 3 times at bat and on Thursday he had 3 hits out of 4 times at bat. What is Logan’s batting average this week? (Important to note that you have fractions but must add num and add den to get to answer and discuss how to tell when this process needs to be used. This will be helpful in the homework)
Adding and Subtracting Fractions
Homework Assignment

Students will be given the following on a sheet to complete for homework. The assignment will be turned in at the next class.

1. Read Case 21 handed out in class and use it to answer the following questions. (Shifter et al., 1999a, pg. 94-97).

2. In case 21, the second class that the case study refers to starts with Ramón representing $\frac{1}{3} + \frac{1}{3}$ this way:

   ![Diagram](image)

   This drawing led him to believe that $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$, but Tanya said “Isn’t that really equal to 2 out of 6 or $\frac{2}{6}$, which is just $\frac{1}{3}$? But how could $\frac{1}{3} + \frac{1}{3} = \frac{1}{3}$?” (Shifter, et al., 1999b, pg 62)
   a. Why did the teacher point out that Ramón’s drawing opened up a mathematical Pandora’s Box?
   b. Explain how Ramón is viewing this problem
   c. Explain how Tanya is viewing this problem
   d. Make up a word problem for each situation.
   e. What does this show you about fractions and adding fractional parts?

3. In trying to sort out the confusion that resulted from Ramón’s drawing, Colin offers another diagram.
   a. What conceptual confusion existed with Colin’s picture?
   b. What could Colin have done differently with his diagrams that would have made a better argument for $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$?

4. In the case study, the teacher decided to throw out the problem $\frac{1}{3} + \frac{1}{3}$ and start with $\frac{1}{3} + \frac{1}{4}$ instead. Lizette’s shares a diagram to help with $\frac{1}{3} + \frac{1}{4}$ which is shown on page 96 of the case study. The class in the case study investigates whether Lizette’s diagram is a good way to come up with the LCM of two numbers.
   a. What do you think – is the diagram helpful for finding the LCM? Why or why not?
   b. Why would the students be discussing LCM when they are talking about adding fractions?

REFERENCES


