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Carlos A. Montes  
*Clemson University*

John C. Ziegert  
*Clemson University*

Chan Wong  
*Clemson University*

Laine Mears  
*Clemson University*, [mears@clemson.edu](mailto:mears@clemson.edu)

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# 2-D ABSOLUTE POSITIONING SYSTEM FOR REAL TIME CONTROL APPLICATIONS

Carlos A. Montes<sup>1</sup>, John C. Ziegert<sup>1</sup>, Chan Wong<sup>1</sup>,  
Laine Mears<sup>1</sup> and Tommy Tucker<sup>2</sup>

<sup>1</sup>Department of Mechanical Engineering  
Clemson University,  
Clemson, SC 29634, USA  
<sup>2</sup>Tucker Innovations Inc.

## INTRODUCTION

The present research introduces a 2-D absolute position feedback method with application to real time position control for manufacturing equipment, such as machine tools. The desired planar displacement command of the tool is produced by an active element or active target on a liquid-crystal display (LCD). A fixed vision sensor is located so that camera plane is parallel to the LCD, which moves freely in the XY plane based upon control action. The sensor observes the motions of the active target and also provides a 2-D coordinate frame based on which in-plane position errors are determined. The machine axes move to reduce these errors causing the planar positioning stage to follow the motions of the target image on the display. Due to the direct-sensing nature of the position transducer no geometric error compensation is required. An image processing algorithm has been developed to retrieve high resolution position and orientation information of a cross-hairs target, displayed on the LCD. This algorithm allows positioning resolution of less than 1/100th of the pixel size on the display. The time delay in the feedback signal to the control system associated with slow hardware frame acquisition rate and long image processing times is addressed through a Smith predictor control scheme. Experimental results are presented.

## TESTBED

The current research is implemented on an XY stage that allows two dimensional motion. An LCD screen is placed upside-down on top of the stage and moves with the stage (Figure 1). The position of the stage is determined through a stationary CCD digital camera that is fixed to the support structure of the stage and aimed at the LCD. A displacement command is displayed on an LCD screen as a moving target image (or active target); then, the position of this target

with respect to a reference point in the image plane (CCD) is estimated. The present work is focused on achieving the desired resolution levels for the proposed platform and fully defining a robust control system structure.

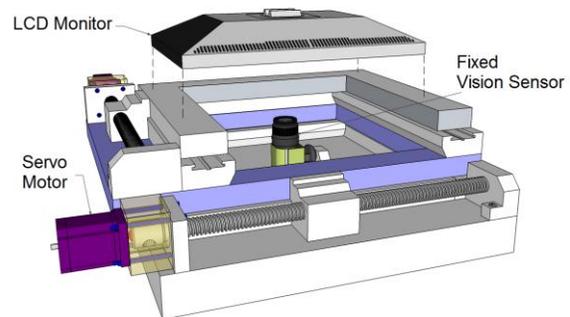


FIGURE 1. 2-axis stage with LCD monitor and fixed CCD camera.

## Cross-Hair Dynamic Target

The target is represented by the intersection of two perpendicular lines displayed on the LCD, i.e. a cross-hairs target. Previous tests have shown that for an 8-bit display, with pixel intensities varying from 0 (lowest, black) to 255 (highest: R, G or B), the optimal configuration is achieved by displaying a high-intensity target over black background (Figure 2 (a)). Aside from the cross-hairs target, four Reference Elements are displayed close to the intersection. These Reference Elements are spaced by known distances on the LCD and are used to estimate the magnification,  $f/Z$ , of a thin-lens perspective projection model, where  $f$  is the focal length and  $Z$  is the out-of-plane distance between the lens and the LCD. If the principal point PP on the CCD (Figure 2 (b)) is regarded as the reference, then an in-plane position error vector can be defined at any time  $t$ , with respect to this reference. As such, a target point  $\mathbf{C}$  on the LCD would have an error vector  $\mathbf{e}(t)=\mathbf{c}$  on the CCD. This vector can be used as the control loop

position error, where the controller's goal is to bring the target to the reference position, i.e. PP.

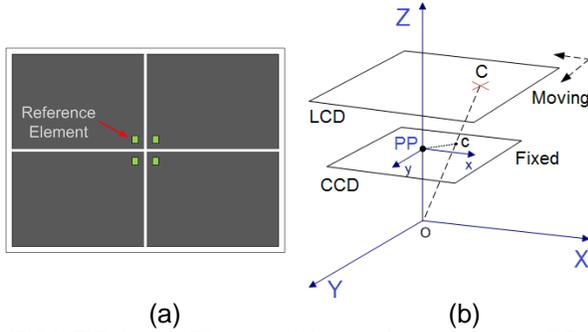


FIGURE 2. (a) Pixelated dynamic target on LCD with four Reference Elements, and (b) pinhole camera model.

## 2-D ABSOLUTE POSITIONING THROUGH NEWTON-RAPHSON ITERATIVE APPROACH

The procedure for locating the target, once captured as a digital image through a CCD monochrome camera, is described in three steps: 1. Fine point location and definition of the line sets; 2. Constrained curve fitting through Newton-Raphson method; and 3. Estimation of pixel magnification using Reference Elements.

### Fine Point Location and Definition of Sets

A fine point location on the CCD is achieved by calculating the intensity weighted center of mass or centroid around an intensity transition. An intensity transition of interest is defined as a vertical or horizontal transition that starts in a black region, goes through a saturation point at 255 and finally goes back to a black region. For a discrete-valued function such as an image,  $I(x,y)$ , where  $x$  varies in a discrete manner over a horizontal array of pixels e.g.  $x \in [0, m-1]$ , and  $y$  is fixed to the current row under analysis e.g.  $y=j$ , the center of mass is

$$X_{C,j} = \frac{\sum_{i=0}^{m-1} x_i I(x_i, j)}{\sum_{i=0}^{m-1} I(x_i, j)} \quad (1)$$

The vertical centroid can be easily calculated by fixing  $x$  to a given column and letting  $y$  vary within a vertical neighborhood of interest. The target orientation, governed by  $\theta$ , is measured with respect to the CCD X-axis and is used to determine the type of transitions present in the target image (Figure 3). If  $|\theta| \leq 15^\circ$  then the

image processing algorithm searches for both horizontal and vertical intensity transitions. Only horizontal transitions are analyzed, otherwise. The threshold value of  $15^\circ$  is selected based on trial and error tests using experimental data. Once the centroids are calculated they are separated in two sets later associated to two different lines.

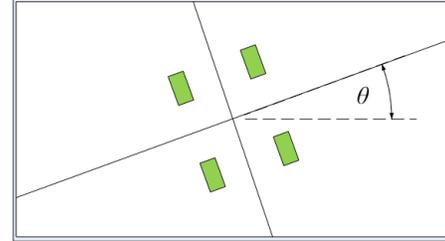


FIGURE 3. Target orientation  $\theta$ .

### Constrained Curve Fitting

The location of the target is obtained by calculating the two lines that best fit the data collected in the previous step, and then analytically computing the intersection of these lines. The two data sets obtained from the previous step are denoted as  $DS1$  and  $DS2$  and are known to contain  $n_1$  and  $n_2$  data points, respectively. The best fit lines are constrained to be perpendicular with respect to each other. The previous statement is equivalent to finding  $\mathbf{y}^{\text{model}}$ , where

$$\mathbf{y}^{\text{model}} = \begin{cases} a_0 + a_1 x, & a_0, a_1 \in \mathbb{R} \text{ and } x \in DS1 \\ a_2 - \frac{1}{a_1} x, & a_2 \in \mathbb{R} \text{ and } x \in DS2 \end{cases} \quad (2)$$

such that

$$S(a_0, a_1, a_2) = \sum_{\text{data}} (\mathbf{y}_i - \mathbf{y}^{\text{model}})^2 = \sum_{DS1} (y_i - a_0 - a_1 x_i)^2 + \sum_{DS2} \left( y_i - a_2 + \frac{1}{a_1} x_i \right)^2 \quad (3)$$

is minimized. The minimum is obtained by finding  $a_0$ ,  $a_1$ , and  $a_2$  such that  $\frac{\partial S}{\partial a_0} = 0$ ,  $\frac{\partial S}{\partial a_1} = 0$  and  $\frac{\partial S}{\partial a_2} = 0$ . From  $\frac{\partial S}{\partial a_0} = 0$ ,  $\frac{\partial S}{\partial a_2} = 0$ , respectively, the following two equations result

$$a_0 = \frac{1}{n_1} (v_2 - a_1 v_1) \quad (4)$$

$$a_2 = \frac{1}{n_2} \left( w_2 + \frac{1}{a_1} w_1 \right) \quad (5)$$

where  $v_1 = \sum_{DS1} x_i$ ,  $v_2 = \sum_{DS1} y_i$ ,  $v_3 = \sum_{DS1} x_i y_i$ ,  $v_4 = \sum_{DS1} x_i^2$  and  $w_i$ ,  $i=1, \dots, 4$ , is equivalent to  $v_i$  but using  $DS2$ . The calculation of  $\frac{\partial S}{\partial a_i} = 0$ , knowing the results from (4) and (5), yields a nonlinear equation of the form  $f(a_i)=0$ , where  $f$  is assumed to have a continuous first derivative  $f'$ . This is

$$f(a_1) = \frac{v_1 v_2}{n_1} - v_3 + a_1 \left( v_4 - \frac{v_1^2}{n_1} \right) + \frac{1}{a_1^2} \left( \frac{w_1 w_2}{n_2} - w_3 \right) + \frac{1}{a_1^3} \left( \frac{w_1^2}{n_2} - w_4 \right) = 0 \quad (6)$$

Equation (6) is solved using Newton-Raphson's iterative method, which implies calculating

$$a_1(n+1) = a_1(n) - \frac{f(a_1(n))}{f'(a_1(n))}, \quad n = 1, 2, \dots, k \quad (7)$$

until

$$|a_1(n+1) - a_1(n)| \leq \varepsilon |a_1(n)| \quad (8)$$

where  $a_i(n)$  represents a constant value given to the variable  $a_i$ ,  $k$  is the total number of iterations,  $\varepsilon$  is an arbitrary threshold and the factor  $|a_i(n)|$  in (8) is required in case of roots of very large or very small absolute value. Once a value for  $a_i$ , satisfying (8), is found (4) and (5) can be calculated and (2) is fully defined. An initial value for  $a_i$  in (7) is determined by applying unconstrained curve fitting to  $DS1$ ; the slope of the unconstrained best fit line is regarded as  $a_i(0)$ . If the criterion diverges for a given target image, unconstrained curve fitting is applied to  $DS2$  as well, and the target is defined as the intersection of the unconstrained curves.

### **Magnification**

The magnification between the LCD- and CCD-pixels is obtained by determining the distance between the four Reference Elements on the CCD and comparing it with the real LCD spacing between these points. The locations of the Reference Elements on the target image are calculated with subpixel resolution using (1) over the areas containing these elements.

### **COMMAND ISSUING**

One LCD pixel consists of three individual stripes (RGB). The intensity of a single LCD-pixel stripe is usually defined in an 8-bit scale. Color intensities on the LCD map to grayscale intensities on the CCD (also in the in an 8-bit scale for an 8-bit camera). Displacement commands are given by changing the intensity weighted centroid on the LCD target pixels, and thereby moving the target by increments in the order of microns. In practice, the camera cannot reliably distinguish between all possibly intensity states of the LCD display. Therefore, a three-tone intensity basis, used to generate horizontal displacement commands, is selected and applies only to those pixels on the target vertical line:  $I_B^X = \{0, 127, 255\}$ . Hence, a displacement basis can be defined:  $D(I_B^X) = \{0.00, \pm 32.58, \pm 49.00\} \mu\text{m}$  (Table 1). From Figure 4 (a) it should be clear that the horizontal displacement basis only needs to cover a 0- to  $\pm 49 \mu\text{m}$  range, starting from a reference stripe  $X_0$ . For displacement bigger than  $|49|$ , a different reference stripe is selected.

TABLE 1. Horizontal centroid change as a function of intensities  $I(0)$  and  $I(1)$ , corresponding to two adjacent stripes and for an LCD pixel size of  $294 \times 292 \mu\text{m}$ .

$I(0)$	$I(1)$	$\Delta X_C (\mu\text{m})$
255	0	0.00
255	127	32.58
255	255	49.00

In order to command horizontal displacements of a fraction of the displacements provided by the basis, some of the LCD pixels on the target vertical line that lie within the camera FOV are set to exhibit a centroid in one location and the rest are set to exhibit a displaced centroid (Figure 4 (b)). For example, if a horizontal displacement command of  $34 \mu\text{m}$  was to be issued, 88% of the pixels in the vertical line would present a centroid at  $32.58 \mu\text{m}$  and 12% would present a displaced centroid at  $49.00 \mu\text{m}$ . Notice that,  $34 \approx 0.88 * 32.58 + 0.12 * 49$ , with an error of less than one micron. For the case of vertical displacement commands a five-tone intensity basis is selected:  $I_B^Y = \{0, 80, 130, 180, 255\}$ . A corresponding vertical displacement basis is defined:  $D(I_B^Y) = \{0.00, \pm 58.26, \pm 82.40, \pm 101.00, \pm 122.00\} \mu\text{m}$

. In general a 2-D displacement command,  $\Delta X_{LCD}$ , must consider the target orientation  $\theta$ . Therefore, a displacement command  $\Delta X_{LCD}$ , displayed on the LCD but reference to the camera plane, requires a displacement  $\Delta \hat{X}_{LCD}$  of the target image referenced to the LCD plane. These two quantities are related through a rotation matrix  $R_\theta$ , i.e.  $\Delta X_{LCD} = R_\theta \Delta \hat{X}_{LCD}$ .

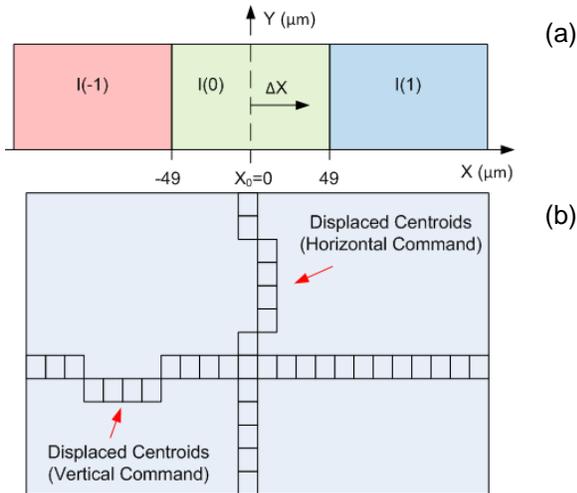


FIGURE 4. (a) One LCD pixel consisting of three stripes: R, G and B. (b) Target on LCD showing a horizontal and a vertical command through displaced centroids.

### CONTROL SYSTEM TIME DELAY

Low camera frame rates and time for image processing cause delays in the acquisition of the computed position of the target image. The delay in the vision-based control system is addressed through a modified version of the well known Smith predictor scheme.

### TEST RESULTS

Experimental data is collected using BMP images. A 100-image sample of the same static target presented a standard deviation of 0.253μm.

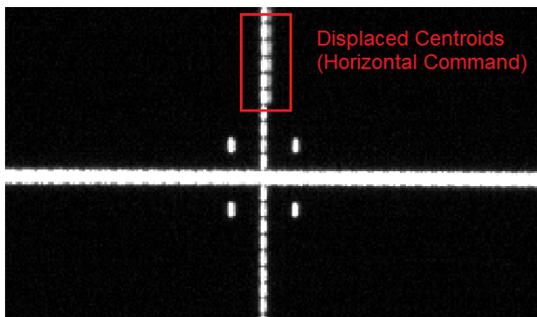


FIGURE 5. Zoom-in experimental target image.

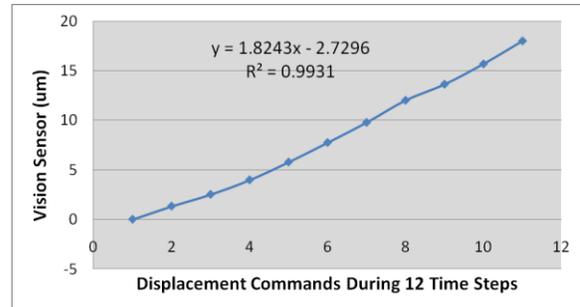


FIGURE 6. 11 horizontal displacement commands of 2μm are represented on the target image. The resulting best fit line has a slope of 1.8243μm/command and a correlation coefficient of 0.9931.

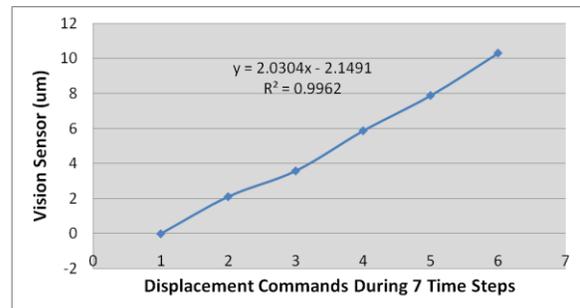


FIGURE 7. 6 vertical displacement commands of 2μm are represented on the target. The resulting best fit line has a slope of 2.0304μm/command and a correlation coefficient of 0.9962.

### CONCLUSIONS

A 2-D position feedback method for real time control applications was presented. Experimental results demonstrate that resolutions on the order of 2μm can be reliably sensed, and displacement commands of the same size can be issued through intensity variations.

### ACKNOWLEDGEMENTS

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