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Optimization Models to Integrate Production and Transportation Planning for Biomass Co-Firing in Coal-Fired Power Plants

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Co-firing biomass is a strategy that leads to reduced greenhouse gas emissions in coal-fired power plants. Incentives such as production tax credit (PTC) are designed to help power plants overcome the financial challenges faced during the implementation phase. Decision makers at power plants face two big challenges. The first challenge is identifying whether the benefits from incentives such as PTC can overcome the costs associated with co-firing. The second challenge is identifying the extent to which a plant should co-fire in order to maximize profits. We present a novel mathematical model that integrates production and transportation decisions at power plants. Such a model enables decision makers evaluate the impacts of co-firing on the system performance and the cost of generating renewable electricity. The model presented is a nonlinear mixed integer program which captures the loss in process efficiencies due to using biomass, a product which has lower heating value as compared to coal; the additional investment costs necessary to support biomass co-firing; as well as savings due to PTC. In order to solve efficiently real-life instances of this problem we present a Lagrangean relaxation model which provide upper bounds and two linear approximations which provide lower bounds for the problem in hand. We use numerical analysis to evaluate the quality of these bounds. We develop a case study using data from nine states located in the southeast region of USA. Via numerical experiments we observe that: (a) Incentives such as PTC do facilitate renewable energy production. (b) The PTC should not be "one size fits all". Instead, tax credits could be a function of plant capacity, or the amount of renewable electricity produced. (c) There is a need for comprehensive tax credit schemes to encourage renewable electricity production and reduce GHG emissions.

Key words: Biomass Co-Firing, Biomass Transportation, Integrated Production Transportation Planning in Supply Chains, Lagrangean Relaxation, Linear Approximation, Nonlinear Programming Model

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1. Introduction

Coal-fired power plants in the US consume 1.1 to 1.2×10^9 tons of coal annually in order to generate electricity. The burning of coal in these plants produces many gases (e.g., CO_2, SO_2, NO_x, etc.) and heavy metals (e.g., mercury and arsenic), which adversely affect the environment and human health (US Energy Information Administration (EIA, 2014)). It is estimated that, for each megawatt-hour of electricity generated, a total of 2,249 lbs of CO_2, 13 lbs of SO_2, and 6 lbs of NO_x are emitted. In 2013, coal accounted for 32% of the total energy-related CO_2 emissions in the US (US Environmental Protection Agency (EPA), 2013).

New performance standards and rules proposed by the EPA have placed stringent limitations on greenhouse gas (GHG) emissions from new and existing power plants. In January 2014, EPA issued a revised performance standard proposal for CO_2 emissions, according to which, new coal fired power plants are required to limit emissions to 1,100 lbs per megawatt-hour. The proposed emissions limit is forcing new coal-fired power plants to identify technologies which will reduce CO_2 emissions by approximately 50%. In June 2014, EPA released proposed rules that are designed to cut CO_2 emissions for existing power plants by 30% from 2005 levels by the year 2030. In March 2013, the agency finalized the Mercury and Air Toxics Standards to reduce emissions of mercury and other air toxics from new and existing coal and oil-fired electric generating units. In July 2011, EPA finalized the Cross-State Air Pollution Rule (CSAPR), which seeks to reduce SO_2 and NO_x emissions from power plants in 28 states.

Researchers agree that co-firing offers a near-term solution to reduce CO_2 emissions from coal-fired power plants since viable and long-term solution alternatives (such as, carbon capture and sequestration (CCS), oxy-firing and carbon loop combustion) still remain in the early to mid stages of development (Basua et al. 2011). Currently, 40 of the 560 coal-fired power plants in the US are co-firing biomass, a renewable energy process that is encouraged by incentives such as the renewable portfolio standards (RPS) at the state level; and the production tax credit (PTC) at the federal level. The existing federal PTC is a flat rate income tax credit of 1.1 cents per kilowatt-hour which supports biomass-based electricity generation technologies such as full-scale biomass co-firing and closed loop partial co-firing; however, its support for general co-firing (open loop biomass) is not clearly specified (Internal Revenue Code, Section 45). The importance of extending current tax incentive plan to cover partial co-firing is suggested in the literature (Smith and Rousaki 2002). In this paper, via our numerical analysis, we evaluate the impacts of extending PTC to support partial co-firing of existing coal-fired power plants in Southeast USA. At the state level, RPS requires investor-owned utilities, electric service providers, and community choice aggregators to increase procurement from eligible renewable energy resources (EIA 2013a, 2013b, 2013c). Researchers also agree that co-firing biomass with coal in power plants is an option for RPS compliance and a near-term solution for introducing biomass into today's renewable energy mix (Basua et al. 2011). Based on the renewable fuel standards (RFS), cellulosic biomass is expected to be the largest source of renewable energy comprising 44.4% of the targets set for 2020 (EPA 2007). Biomass used in direct combustion has shown to be dispatchable, i.e., capable of responding to user needs without energy storage unlike wind and solar power which are at the mercy of nature (Tillman et al. 2010). While the technology to produce liquid fuels by using biomass is not yet available, co-firing of biomass is a feasible option worth investigating.

Based upon our review of the literature, we contend that most current research has involved elucidating the technological aspects of co-firing processes (Li et al. 2012, Tumuluru et al. 2012) and the techno-economic and feasibility analysis (Dong et al. 2010, Ruhul-Kabir and Kumar 2012, Steer et al. 2013, Goerndt et al. 2013b, Paudel 2013, Mehmood et al. 2014). Very little research has been undertaken to estimate the transportation costs of delivering biomass to power plants (Roni et al. 2014). To the best of our knowledge, there are no studies which provide models to optimize co-firing decisions at the plant level by integrating plant operations-, with, transportation and other logistics-related decisions. Thus, the main contribution of this paper is the development of mathematical models to aid co-firing decisions at plant level. The proposed model takes a holistic view of the processes affected by these decisions such as production, storage, and transportation.

Biomass co-firing impacts the performance of the coal plants in several ways. *First*, biomass has less energy density as compared to coal, and therefore, larger quantities of biomass are required to substitute the same amount of coal. Additionally, biomass in the form of agricultural and forest waste has poor flowability properties, and thus, it is bulky, heterogeneous, and unstable. For these reasons, processes such as loading, unloading and transportation of biomass are challenging and expensive. *Second*, existing power plants are typically co-located with coal mines which would typically supply enough coal to satisfy plant's demand. Biomass suppliers are typically small or medium sized farms, which are widely dispersed geographically. Thus, processes such as biomass collection, biomass delivery, and supplier management are expensive (Aden et al. 2002). *Third*, co-firing of biomass reduces boilers efficiency, and as a consequence, reduces overall system efficiency. *Fourth*, biomass co-firing requires investments to adjust the feeding system, since the same system often cannot be used to feed biomass in burners (Tillman 2000).

Coal plants are aware of the challenges and opportunities related to co-firing. However, decision makers are in need of tools which integrate the additional savings, additional costs, and loss of process efficiencies from co-firing. Such tools would enable decision makers identify the level of co-firing that maximizes profits while complying with existing GHG emission regulations. To support these decisions we propose an optimization model which encompasses the (a) additional investments necessary to adopt co-firing at power plants; (b) reduction in process and equipment (e.g., boiler) efficiency from this coal substitution; (c) additional transportation-related costs necessary for biomass delivery; (d) savings from incentives such as PTC. This model is useful in evaluating the existing trade-offs between profits and the environmental impacts associated with co-firing. We propose solution approaches to solve these large scale, nonlinear optimization problems. These approaches are novel and rely on the properties of the models presented.

Another important contribution of this paper is that we use real world data to build a case study. Thus, through our numerical analysis we make a few important observations about the impact of incentives such as PTC on renewable electricity production. These findings can help policy makers at the federal or state level to evaluate the economic feasibility of producing renewable electricity, and design policies in support of co-firing.

2. Review of Related Literature

The work presented in this paper contributes to the literature on biomass supply chain optimization as well as technological and economical feasibility of co-firing.

2.1. Biomass supply chain optimization

The literature on biomass supply chain has grown in the recent years. Early studies in the area of biomass supply chain management focused mainly on cost-benefit analysis, such as estimating the cost of collecting, handling, and hauling biomass (Perlack and Turhollow 2002, Petrolia 2008) and comparing different modes of transportation to deliver biomass (Kumar et al. 2005, Mahmudi and Flynn 2006). This literature pays attention to mainly operational-level supply chain decisions. More recently, a number of models have been proposed to optimize the performance of the supply chain by incorporating strategic and tactical decisions. Models proposed by Eksioglu et al. (2009), Zamboni et al. (2009), Huang et al. (2010), An et al. (2011) integrate plant location, production, and transportation decisions in the biomass supply chain. For a comprehensive review of modeling frameworks, challenges faced, and the future of biomass supply chains we refer the readers to Sharma et al. (2013).

Related to this research are works by Aguilar et al. (2012) and Roni et al. (2014). Aguilar et al. (2012) propose a supply chain model to evaluate the likelihood of using biomass for co-firing. The model evaluates the impact of the locations of biomass suppliers and the location of coal-fired power plant on co-firing decisions. Roni et al. (2014) propose a framework to design biomass supply chains to support co-firing of biomass at the national level. Based on this framework, small-sized plants are better off receiving biomass shipments from local suppliers. Large-sized plants are better-off using hub-and-spoke in-bound networks that rely both on truck and rail transportation for biomass delivery. Hub-and-spoke networks are typically used for long-haul delivery of bulky products. These models either focus on optimizing transportation decisions for a given biomass co-firing strategy or focus on optimizing co-firing decisions within a plant given the amount of biomass available in the region. The model we propose integrates transportation and co-firing decisions with the goal of optimizing system-wide profits.

2.2. Technological and economical feasibility of co-firing

Most of the literature about co-firing is mainly focused on analyzing its technological and economical feasibility. Work by Goerndt et al. (2013a) identifies the necessary drivers for successful implementation of co-firing. The drivers identified are the adequate biomass supply and competitive biomass purchase and transportation costs. The work of Baxter (2005) indicates that biomass-coal co-combustion is an affordable renewable energy option that promises reductions in GHG emissions. Works of Hansson et al. (2009) and Al-Mansour and Zuwala (2010) indicate that biomass co-firing is a technologically sound and near-term solution to comply with GHG emission regulations in the European Union (EU). They support their findings by discussing some successful implementations of the technology in EU. A study by Basua et al. (2011) indicates that nearly all coal-fired power plants can achieve an incremental gain in GHG reductions with minimum modifications and moderate investments. Hansson et al. (2009) predict that biomass co-firing will become a major contributor to meeting the renewable energy production goals in near future. Works by Li et al. (2012), Shao et al. (2012), Tumuluru et al. (2012), Steer et al. (2013), Tchapda and Pisupati (2014) investigate the technological challenges and process inefficiencies associated with biomass co-firing.

Baxter (2005), De and Assadi (2009), Wils et al. (2012), O'Mahoney et al. (2013), Paudel (2013) study the economic feasibility of co-firing. Ruhul-Kabir and Kumar (2012) conduct a life cycle energy and environmental performance analysis of co-firing different types of biomass since the efficiency of co-firing process depends on the specific chemical content and properties of the biomass used in co-combustion (Mehmood et al. 2014). O'Mahoney et al. (2013) and Wils et al. (2012) use a cost-benefit analysis to show that governmental incentives are necessary for making co-firing an attractive investment option. Similarly, McIlveen-Wright et al. (2011) and De and Assadi (2009) conduct comprehensive techno-economic analysis of co-firing. They evaluated the technological and economical feasibilities of existing pilot plants, and suggest that there is a need for additional governmental incentive schemes. The effect of subsidizing biomass co-firing is also discussed by Lintunen and Kangas (2010). Their numerical results show that subsidizing biomass combustion in a coal-fired power plant provides great results with minimum investments in renewable technology. Tharakan et al. (2005) evaluate the impacts of three co-firing incentive programs in the US. One of the incentives analyzed is the PTC.

3. Problem Description

There are two main co-firing methods used in coal plants, which are direct and indirect co-firing. Direct biomass co-firing systems include solutions such as: co-milling, co-feeding, combined burner and new burners. In these systems, biomass is milled and then fed to coal burners for combustion.

This method is the simplest, cheapest and most-widely used (see Touš et al. (2011), Piriou et al. (2013)). However, direct co-firing is sensitive to the biomass quality, and, in the long run, direct co-firing shortens the lifespan of equipment used. Indirect biomass co-firing systems include solutions such as: separated burning, coupled plant, gasification systems, and pyrolysis. In these systems, biomass is either burned separately using specially designed boilers; or, it is transformed into a gas using a gasifier; or it is transformed into a mixture of gas, bio-oils and char through pyrolysis (see Dong et al. (2010), Caputo et al. (2005), Dasappa et al. (2004)). These systems are more complex and expensive. However, these systems reduce equipment degradation problems, such as, corrosion, fouling, and slagging. Such systems allow for larger co-firing rates as compared to direct co-firing.

The focus of this study is direct co-firing since this method is easy to implement, requires less capital investments, thus, easier to adopt by existing coal fired power plants. In this case, the percentage of coal substituted varies between 0-50%.

3.1. Biomass Co-Firing: Modeling Plant Efficiency

Biomass has a lower heating value as compared to coal. Additionally, using biomass negatively impacts the efficiency of the burners used in a coal plant. Thus, co-firing as much biomass (by mass) as the amount of coal displaced would reduce the amount of energy generated. The objective of this section is to determine the relationship that exists between the amount of coal displaced and the amount of biomass co-fired to maintain the same energy output at a coal plant.

Let Q_j^0 (in MW) be the initial (before co-firing) annual heat input rate of a coal plant j. The heat input is a function of plant's nameplate capacity (TC_j in MW), capacity factor f_j (or utilization rate), and initial plant efficiency rate (ρ_j^0). The annual heat input is equal to $Q_j^0 = \frac{TC_j * f_j}{\rho_j^0}$.

A coal plant would typically use coal in order to generate electricity. The mass of coal used is a function of the lower heating value for coal (LHV_j^{coal} in BTU/ton) and the total number of operating hours (OH_j in hours/year). The amount of coal used (M_j^{coal} in tons) is equal to:

$$M_j^{coal} = \frac{Q_j^0 * OH_j * C^{wb}}{LHV_j^{coal}}, \tag{1}$$

where, C^{wb} is the conversion factor from 1 MW to BTU/hr.

Suppose that ∇M_j^{coal} tons of coal will be displaced in the coal plant. We estimate the amount of biomass required (M_j^{bm} in tons) to maintain the same energy output using the following energy equilibrium equation

$$M_j^{bm} * LHV_j^{bm} = \nabla M_j^{coal} * LHV_j^{coal}.$$

Thus, the amount of biomass required to displace ∇M_j^{coal} tons of coal is equal to

$$M_j^{bm} = \nabla M_j^{coal} * \left(\frac{LHV_j^{coal}}{LHV_j^{bm}} \right).$$

We now can calculate β_j, the percentage of biomass co-fired in facility j, as follows:

$$\beta_j = \frac{M_j^{bm}}{(M_j^{coal} - \nabla M_j^{coal}) + M_j^{bm}} = \frac{1}{\frac{M_j^{coal}}{M_j^{bm}} + (1 - \frac{LHV_j^{bm}}{LHV_j^{coal}})} = \frac{1}{\frac{M_j^{coal}}{M_j^{bm}} + \alpha_j}, \tag{2}$$

where $\alpha_j = 1 - \frac{LHV_j^{bm}}{LHV_j^{coal}}$. Thus, for a fixed value of β_j, the amount of biomass required to displace coal should be:

$$M_j^{bm} = \frac{M_j^{coal}}{1/\beta_j - \alpha_j} = \left(\frac{1}{1/\beta_j - \alpha_j}\right) * \left(\frac{Q_j^0 * OH_j * C^{wb}}{LHV_j^{coal}}\right). \tag{3}$$

Equation (3) calculates the amount of biomass required to displace $\beta\%$ of coal under the assumption that there would be no equipment efficiency loss due to co-firing. However, equipment efficiency is indeed affected. Let ρ_j denote plant efficiency, which is a function of the efficiency of all processes involved. Initially, $\rho_j^0 = \rho_j^b * \rho_j^{rp}$, where, ρ_j^b represents boiler efficiency and ρ_j^{rp} represents the efficiency of rest of the plant. The efficiency loss of boilers (EL_j) due to displacing $\beta_j\%$ of coal, is calculated as follows: $EL_j = 0.0044\beta_j^2 + 0.0055$ (Tillman 2000). Due to this efficiency loss, plant efficiency decreases from ρ_j^0 to $\rho_j = (\rho_j^b - EL_j) * \rho_j^{rp}$.

The efficiency loss impacts the annual heat input of the coal plant. Thus, the heat input required to maintain the same energy output increases to:

$$Q_j = \frac{TC_j * f_j}{\rho_j} = \left(\frac{\rho_j^0}{\rho_j}\right) * Q_j^0 = \left(\frac{\rho_j^b}{\rho_j^b - EL_j}\right) * Q_j^0.$$

Consequently, the corresponding amount of biomass required for co-firing increases to:

$$M_j^{bm} = \left(\frac{1}{1/\beta_j - \alpha_j}\right) * \left(\frac{Q_j^0 * OH_j * C^{wb}}{LHV_j^{coal}}\right) * \left(\frac{\rho_j^b}{\rho_j^b - EL_j}\right) =$$
$$= \left(\frac{1}{1/\beta_j - \alpha_j}\right) * \left(\frac{\rho_j^b}{\rho_j^b - EL_j}\right) * M_j^{coal}. \tag{4}$$

Equation (4) indicates that the amount of biomass requirement to displace $\beta_j\%$ of the coal is a function of plant nameplate capacity, plant efficiency, lower heating values of coal, lower heating values of biomass, and plant operating hours.

3.2. Biomass Co-Firing: Modeling Costs and Savings

This section estimates the additional costs and savings due to biomass co-firing.

Plant Investment Costs: Investments on building a new feeding system, purchasing compressors and dryers, purchasing biomass handling equipment, and investments on additional storage space are typically required to facilitate direct co-firing. Studies such as, Sondreal et al. (2001), Caputo et al. (2005) indicate that when less than 4% of coal is displaced in a plant, the existing fuel feeding system can be used for both products. In this case, the annual investments (I_j^{CAP})

of plant j are expected to be \$50 per KW of power generated from biomass, assuming 20 years investment lifetime and 9% discount rate.

In order to calculate the annual investment costs, we first need to calculate how much power (in MW) could be generated from biomass at a plant of capacity TC_j when $\beta_j\%$ of coal ($\beta_j\% < 4\%$) is being displace. Next, we multiply this amount with the \$50/KW (or \$50,000/MW) to calculate the annual investment costs as follows:

$$I_j^{CAP} = 50,000 * \left(\frac{M_j^{bm}}{M_j^{coal} - \nabla M_j^{coal}}\right) * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right) = 50,000 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right) * \left(\frac{\beta_j}{1-\beta_j}\right).$$

Let, $I_j^{cap} = 50,000 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)$, then,

$$I_j^{CAP} = I_j^{cap} * \left(\frac{\beta_j}{1-\beta_j}\right). \tag{5}$$

In the case when $\beta_j > 4\%$, the annual investment costs are higher since large amounts of biomass would be used by the plant. In this case, the plant would be investing in extra storage space, material handling equipments, and compressors and dryers necessary to process biomass prior to co-firing. The annual investment costs necessary for biomass storage (I_j^S), biomass handling (I_j^H), and investments on compressors and dryers (I_j^{CD}) are presented next. The annual storage costs are estimated to be \$136,578 per MW of power generated from biomass; the annual handling costs are estimated to be \$55,780 and the annual compressors and dryers costs are \$13,646 per MW of power generated from biomass (Caputo et al. 2005).

The annual cost of biomass storage as follows:

$$I_j^S = 136,578 * \left(\frac{M_j^{bm}}{M_j^{coal} - \nabla M_j^{coal}} * TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.5575} =$$
$$136578 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}} * \frac{\beta_j}{1-\beta_j}\right)^{0.5575}.$$

The annual cost of biomass handling is estimated as follows:

$$I_j^H = 55780 * \left(\frac{M_j^{bm}}{M_j^{coal} - \nabla M_j^{coal}} * TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.9554} =$$
$$55780 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}} * \frac{\beta_j}{1-\beta_j}\right)^{0.9554}.$$

The annual cost of compressors and dryers is estimated as follows:

$$I_j^{CD} = 13646 * \left(\frac{M_j^{bm}}{M_j^{coal} - \nabla M_j^{coal}} * TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.5575} =$$

$$13646 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}} * \frac{\beta_j}{1-\beta_j}\right)^{0.5575}.$$

Let, $I_j^s = 136578 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.9554}$, $I_j^h = 55780 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.9554}$, and $I_j^{cd} = 13646 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.5575}$.

The total capital investment costs in plant j when $\beta_j \geq 4\%$ are:

$$I_j^{CAP} = I_j^S + I_j^H + I_j^{CD} = I_j^s \left(\frac{\beta_j}{1-\beta_j}\right)^{0.5575} + I_j^h \left(\frac{\beta_j}{1-\beta_j}\right)^{0.9554} + I_j^{cd} \left(\frac{\beta_j}{1-\beta_j}\right)^{0.5575}. \tag{6}$$

Operating Costs: Operating costs consist of the cost of purchasing and transporting biomass. Let c_i^{bm} denote the unit purchase cost of biomass (in \$/ton) from supplier i, and, let S denote the set of biomass suppliers. Then, the total biomass purchasing cost at plant j is equal to $\sum_{i \in S} c_i^{bm} * M_j^{bm}$.

Transportation costs consist of the trucking costs necessary to deliver biomass to coal plants. We assume that truck shipments of biomass are delivered by third party service providers who charge a fixed \$ amount per ton of biomass shipped. The unit delivery cost from supplier i to plant j is denoted by c_{ij}. The total biomass transportation costs of plant j are equal to $\sum_{i \in S} c_{ij} M_j^{bm}$.

Savings: Savings resulted from the PTC of 1.1¢ per KWh of renewable electricity, and from the displacement of ∇M_j^{coal} tons of coal.

Savings due to the PTC are calculated as follows:

$$S_j^{tax} = \sigma_j^t * M_j^{bm}, \tag{7}$$

where, $\sigma_j^t = 11 * \frac{LHV_j^{bm}}{C^{wb}}$.

Savings due to coal displacement are calculated as follows:

$$S_j^p = c_j^{coal} * (\nabla M_j^{coal}) = c_j^{coal} * \left(M_j^{bm} * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right) = \sigma_j^p * M_j^{bm}, \tag{8}$$

where, $\sigma_j^p = c_j^{coal} * \frac{LHV_j^{bm}}{LHV_j^{coal}}$. Here c_j^{coal} is the door price of coal (in \$/ton). This cost includes purchasing and transportation costs.

4. A Mixed Integer Nonlinear Programming Formulation

This section presents a nonlinear problem formulation which identifies co-firing strategies that optimize the total profits of coal-fired power plants which share the same regional biomass resources. The model presented is a nonlinear mixed-integer program. In the following sections we present a Lagrangean relaxation algorithm that generates upper bounds for the non-linear model; as well as two linear approximation that provide feasible solutions to the nonlinear model.

Let X_{ij} be a decision variable which represents the amount of biomass (in tons) delivered annually from supplier i to coal plant j. Let B_j be a decision variable which represents the percentage of coal displaced in plant j. Let C denote the set of coal plants, and S denote the set of suppliers in the supply chain. Then, the amount of biomass used in plant j can be represented as

$$M_j^{bm} = \sum_{i \in S} X_{ij}.$$

We use Equation (4) to derive the following expression which represents the amount of biomass used as a function of the decision variables declared.

$$\sum_{i \in S} X_{ij} = \left(\frac{1}{1/B_j - \alpha_j}\right) * \left(\frac{\rho_j^b}{\rho_j^b - 0.0044B_j^2 - 0.0055}\right) * M_j^{coal}. \tag{9}$$

We express the savings from biomass co-fire at plant j as a function of these decision variables as follows:

$$\sum_{i \in S} (\sigma_j^p + \sigma_j^t) X_{ij}.$$

Biomass purchasing costs at plant j are equal to

$$\sum_{i \in S} c_i^{bm} X_{ij}.$$

Truck transportation costs at plant j are equal to

$$\sum_{i \in S} c_{ij} X_{ij}.$$

As described in Section 3.2, the functions used to estimate investment costs for $B_j\% < 4\%$ are different from the functions used when $B_j\% \geq 4\%$. In order to capture these differences in our model, we introduce the binary decision variables Y_j, and define them as follows:

$$Y_j = \begin{cases} 1 & \text{if } B_j \leq 0.04 \\ 0 & \text{if } B_j > 0.04 \end{cases}$$

We linearize the relationship between Y_j and B_j using the following equations.

$$B_j \leq 0.04 + M(1 - Y_j)$$
$$B_j > 0.04 * (1 - Y_j)$$

We can now express the investment costs of plant j as:

$$I_j^{cap} * \left(\frac{B_j}{1 - B_j}\right) Y_j + \left(I_j^s + I_j^{cd}\right) \left(\frac{B_j}{1 - B_j}\right)^{0.9554} (1 - Y_j) + I_j^h * \left(\frac{B_j}{1 - B_j}\right)^{0.9554} (1 - Y_j).$$

The following is the nonlinear mixed-integer programming formulation for this problem which we will be referring to as formulation (**P**).

$$Maximize: \mathcal{Z}^P(X,Y,B) = \sum_{j\in C}\left(\sigma_j^p + \sigma_j^t\right)\left(\sum_{i\in S} X_{ij}\right) - \sum_{i\in S}\sum_{j\in C}(c_{ij} + c_i^{bm})X_{ij} -$$
$$-\sum_{j\in C} I_j^{cap}\left(\frac{B_j}{1-B_j}\right)Y_j - \sum_{j\in C} I_j^h\left(\frac{B_j}{1-B_j}\right)^{0.9554}(1-Y_j) -$$
$$-\sum_{j\in C}\left(I_j^s + I_j^{cd}\right)\left(\frac{B_j}{1-B_j}\right)^{0.5575}(1-Y_j)$$

Subject to:

$$\sum_{j\in C} X_{ij} \le s_i \qquad \forall i \in S, \tag{10}$$
$$\sum_{i\in S} X_{ij} \le \frac{(M_j^{coal} * \rho_j^b)}{(1/B_j - \alpha_j)(\rho_j^b - 0.0044B_j^2 - 0.0055)} \qquad \forall j \in C, \tag{11}$$
$$B_j \le 0.04 + M(1-Y_j) \qquad \forall j \in C, \tag{12}$$
$$B_j > 0.04(1-Y_j) \qquad \forall j \in C, \tag{13}$$
$$X_{ij} \in R^+ \qquad \forall i \in S, j \in C \tag{14}$$
$$B_j \in [0,1] \qquad \forall j \in C. \tag{15}$$
$$Y_j \in \{0,1\} \qquad \forall j \in C. \tag{16}$$

The objective function maximizes the benefits of co-firing across all $j \in C$. Constraints (10) indicate that the biomass delivered by supplier i is limited by its availability (s_i). Constraints (11) represent the amount of biomass required in a plant as a function of plant capacity, plant efficiency and as a function of the percentage of biomass co-fired. Constraints (12) and (13) provide a linear representation of the relationship between the decision variables B_j and Y_j. Constraints (14) are the non-negativity constrains, (15) are the boundary constraints, and (16) are the binary constraints.

5. Generating Upper Bounds via a Lagrangean Relaxation Algorithm

In this section we present a Lagrangean relaxation algorithm that generates upper bounds for model (**P**). This algorithm relaxes constraints (10). The Lagrangean relaxation model is:

$$Maximize: \mathcal{Z}^P(\lambda) = \sum_{i\in S}\sum_{j\in C}(\overline{c}_{ij} - \lambda_i)X_{ij} + \sum_{i\in S} s_i\lambda_i - \sum_{j\in C} I_j^{cap}\left(\frac{B_j}{1-B_j}\right)Y_j$$
$$-\sum_{j\in C} I_j^h\left(\frac{B_j}{1-B_j}\right)^{0.9554}(1-Y_j) - \sum_{j\in C}\left(I_j^s + I_j^{cd}\right)\left(\frac{B_j}{1-B_j}\right)^{0.5575}(1-Y_j)$$

Subject to: (11) to (16)

Where, $\bar{c}_{ij} = \sigma_j^p + \sigma_j^t - c_i^{bm} - c_{ij}$. The Lagrangean dual (**LD**) problem is: $\min_{\lambda\ge 0} \mathcal{Z}^P(\lambda)$.

The Lagrangean relaxation model $\mathcal{Z}^P(\lambda)$ can be decomposed into $|C|$ single plant problems. We refer to the single plant problems as subproblems $(\mathbf{SP})_j$. The following is the corresponding problem formulation.

$$Maximize: \mathcal{Z}^{SP_j}(X,Y,B) = \sum_{i \in S} \overline{c}_i X_i - I^{cap}\left(\frac{B}{1-B}\right)Y$$
$$-I^h\left(\frac{B}{1-B}\right)^{0.9554}(1-Y) - \left(I^s + I^{cd}\right)\left(\frac{B}{1-B}\right)^{0.5575}(1-Y)$$

Subject to:

$$X_i \leq s_i \qquad \forall i \in S \qquad (17)$$
$$\sum_{i \in S} X_{ij} \leq \frac{(M_j^{coal} * \rho_j^b)}{(1/B_j - \alpha_j)(\rho_j^b - 0.0044B_j^2 - 0.0055)} \qquad (18)$$
$$B \leq 0.04 + M(1-Y) \qquad (19)$$
$$B > 0.04(1-Y) \qquad (20)$$
$$X_i \in R^+ \qquad \forall i \in S \qquad (21)$$
$$B \in [0,1] \qquad (22)$$
$$Y \in \{0,1\} \qquad (23)$$

Constraints (17) are valid inequalities since each feasible solution to the single plant problem $(\mathbf{SP})_j$ meets these supply limitation constraints. The single plant problem can further be decomposed into three sub-problems. Subproblem 1 assumes $B^* \in [0, 0.04]$, subproblem 2 assumes $B^* \in (0.04, 0.221]$, and subproblem 3 assumes $B^* \in (0.221, 0.5]$.

Subproblem 1:

$$Maximize: \mathcal{Z}(X,B) = \sum_{i \in S} \overline{c}_i X_i - I^{cap}\left(\frac{B}{1-B}\right)$$

Subject to: (17), (18), (21)

$$B \in [0, 0.04]$$

Subproblem 2:

$$Maximize: \mathcal{Z}(X,B) = \sum_{i \in S} \overline{c}_i X_i - I^h\left(\frac{B}{1-B}\right)^{0.9554} - \left(I^s + I^{cd}\right)\left(\frac{B}{1-B}\right)^{0.5575}$$

Subject to: (17), (18), (21)

$$B \in (0.040.221]$$

Subproblem 3:

$$Maximize: \mathcal{Z}(X,B) = \sum_{i\in S}\overline{c}_i X_i - I^h\left(\frac{B}{1-B}\right)^{0.9554} - \left(I^s + I^{cd}\right)\left(\frac{B}{1-B}\right)^{0.5575}$$

$$\text{Subject to: (17), (18), (21)}$$

$$B \in (0.2210.5]$$

These subproblems are easy and can be solved by inspection in polynomial time. The corresponding algorithm is presented in Table 8.

Theorem 1. *In an optimal solution $X^* = \{X_1^*, X_2^*, \ldots, X_l^*\}$ to the single plant problem, at most one of the suppliers' is used partially. That means, $X_i^* = s_i$, $\forall i \in S^*/k$; $X_k^* = \gamma$; and $X_i^* = 0$ for $i \notin S^*$. Where, S^* is the set of suppliers selected in the optimal solution. (See the proof in Appendix A.)*

Theorem 2. *The special case of problem ($\mathbf{P}$) -when there is a single plant in the supply chain- can be solved to optimality via an $O(|S|log(|S|))$ algorithm, where $|S|$ represents the number of suppliers in the supply chain. (See the proof in Appendix A.)*

The algorithm that solves the single plant problem starts by sorting the suppliers in a decreasing order of $\overline{c}_i$. Without loss of generality, we assume that $\overline{c}_i > 0$ for $i \in S$. Let S^* denote the set of suppliers selected in an optimal solution. Initially S^* is empty. We start by finding the B_1^* that maximizes $\mathcal{Z}(X,B)$ for $B \in [0, 0.04]$. Next, we find B_2^* that maximizes $\mathcal{Z}(X,B)$ for $B \in (0.04, 0.221]$, and B_3^* that maximizes $\mathcal{Z}(X,B)$ for $B \in (0.221, 0.5]$. Therefore, the optimal solution to this problem is $B^* = \arg\max\{\mathcal{Z}(B_1^*), \mathcal{Z}(B_2^*), \mathcal{Z}(B_3^*)\}$. Recall that, in an optimal solution constraints (18) are binding. Thus, we can express the optimal objective function value as a function of B only.

Let's show how we find B_1^*. We start with supplier 1 and calculate $\mathcal{Z}(B_1)$. If $\mathcal{Z}(B_1) > 0$, then $S^* = S^* \bigcup 1$. We add supplier i to S^* as long as the following holds true $Z\mathcal{Z}(B_{i-1}) < \mathcal{Z}(B_i) > 0$. Let supplier j be such that $\mathcal{Z}(B_{j-2}) < \mathcal{Z}(B_{j-1}) > \mathcal{Z}(B_j)$. This implies that at some $B^* \in [B_{j-1}, B_j]$ function $\mathcal{Z}(B)$ reached its maximum. Since the slope of $\mathcal{Z}(B)$ could change its sign at most twice in the interval $[B_{j-1}, B_j]$, and $\mathcal{Z}(B_{j-1}) > \mathcal{Z}(B_j)$, that means, within this interval, the slope increased from B_{j-1} to $\overline{B} \leq B_j$, and then, decreased from $\overline{B}$ to B_j. This implies that there is at most one maximum within this interval. We use the Golden Search algorithm to identify B^* which maximized $\mathcal{Z}(B)$ (Luenberger and Ye 2008). The Golden Search algorithm will be used at most three times, ones for each interval $[0, 0.04], [0.04, 0.221), (0.221, 0.5]$.

Figure 5 outlines the Lagrangean relaxation algorithm. In each iteration of this algorithm $|C|$ single plant problems are solved. These solutions are used to update the upper bound (UB).

The lower bound is found by solving model (**Q**) (see Section 6.1). We employ the subgradient optimization method to solve the Lagrangean dual problem (**LD**) (Nemhauser and Wolsey 1988) and update the Lagrangean multipliers λ_i. We use the following equation:

$$\lambda_i^n = \lambda_i^{n-1} + u^n(s_i - \sum_j X_{ij}^n),$$

where $u^n = \frac{\xi^n(UB-LB)}{\sum_{i\in S}(s_i - \sum_j X_{ij})}$. The parameter $\xi \in (0;2]$ is reduced if the upper bound fails to improve after a fixed number of iterations. The algorithm stops if one of the following conditions is satisfied: (i) the error gap ($\epsilon = \left(\frac{UB-LB}{LB}\right) * 100$) is less than 1%, or (ii) the number of iterations reaches a pre-specified bound.

6. Generating Lower Bounds via Linear Approximation Algorithms

6.1. A Linear Mixed Integer Problem Formulation

Let's assume that plant j decides to use biomass to displace coal at a fix rate of $\beta_j = 1\%$, 2%, 3% etc. Without loss of generality, we assume that this plant would pursue a single coal displacement strategy, and therefore, would select a single value of β_j. We denote the finite set of all the values that β_j can potentially take by L. Let $l = 1, \ldots, |L|$ index this set, and let L_l denote the $l-$the element of this set. We declare Y_{lj} to be a binary variable which takes the value 1 if facility j displaces $L_l = \beta_j\%$ coal, and takes the value 0 otherwise.

For a given value of β_j, the amount of biomass needed at plant j is constant and is calculated using equation (4). We denote this amount by M_{lj}^{bm}. The total amount of biomass required at plant j is equal to:

$$M_j^{bm} = \sum_{l\in L} M_{lj}^{bm} Y_{lj}. \tag{24}$$

Investment costs also depend on the value of β_j. For a given value of β_j these costs are fixed. Thus, we use equation (5) to calculate investment costs when $\beta_j \leq 0.04$. For $\beta_j > 0.04$, we calculate investment costs using equation (6). Let I_{lj} denote investment costs at plant j for a given value of β_j. The total investment costs are equal to

$$\sum_{l\in L}\sum_{j\in C} I_{lj} Y_{lj}. \tag{25}$$

The following is a linear mixed integer programming formulation for problem (**P**) which we will be referring to as formulation (**Q**).

$$Maximize: \mathcal{Z}^Q(X,Y) = \sum_{i\in S}\sum_{j\in C} \bar{c}_{ij} X_{ij} - \sum_{l\in L}\sum_{j\in C} I_{lj} Y_{lj}$$

Subject to:

$$\sum_{j\in C} X_{ij} \leq s_i \qquad \forall i \in S, \tag{26}$$

$$\sum_{l\in L} Y_{lj} \leq 1 \qquad \forall j \in C, \tag{27}$$

$$\sum_{i\in S} X_{ij} \leq \sum_{l\in L} M_{lj}^{bm} Y_{lj} \qquad j \in C, \tag{28}$$

$$X_{ij} \in R^+ \qquad \forall i \in S, j \in C \tag{29}$$

$$Y_{lj} \in \{0,1\} \qquad \forall l \in L, \forall j \in C \tag{30}$$

The objective function maximizes the benefits of co-firing across all $j \in C$. Constraint (27) limits the number of co-firing strategies adopted by a coal plant to one. Constraints (28) set the upper bound on the amount of biomass requirements based on the co-firing strategy selected. (29) and (30) are the non-negativity and binary constraints.

Proposition 1: *A feasible solution to problem (*$\mathbf{Q}$*) is feasible to the non-linear problem (*$\mathbf{P}$*); and the objective function value of (*$\mathbf{Q}$*) is a lower bound for problem (*$\mathbf{P}$*). (See the proof in Appendix B.)*

Theorem 3. *As $|L|$ approaches infinity, an optimal solution to (*$\mathbf{Q}$*) is optimal to (*$\mathbf{P}$*) with probability 1. (See proof in Appendix B.)*

6.2. A Linear Approximation of Model (P)

Linearizing constraints (11):

The right hand side of constraints (11) are nonlinear functions. Let $f_j = \frac{(\rho_j^b * B_j)}{(1-\alpha_j * B_j)(\rho_j^b - 0.0044B_j^2 - 0.0055)}$. Thus, these constraints can be expressed as:

$$\sum_{i\in S} X_{ij} \leq M_j^{coal} * f_j \qquad \forall j \in C.$$

Proposition 2:

$$B_j \leq \frac{1}{M_j^{coal}} \sum_{i\in S} X_{ij} \leq f_j \leq (B_j + \overline{a}_j) \qquad \forall j \in C.$$

Where, $\overline{a}_j = \frac{0.5\left(\rho_j^b - (1-0.5\alpha_j)(\rho_j^b - 0.0066)\right)}{(1-0.5\alpha_j)(\rho_j^b - 0.0066)}$ (See proof in Appendix B).

Corollary 1: *Let (*$\mathbf{LR}$*) be the following linear approximation of problem (*$\mathbf{P}$*).*

$$Maximize: \quad \mathcal{Z}^P(X,Y,B)$$

Subject to: (10), (12) to (16)

$$M_j^{coal} * B_j \leq \sum_{i\in S} X_{ij} \leq M_j^{coal} * (B_j + \overline{a}_j) \qquad \forall j \in C. \tag{31}$$

Problem (**LR**) *is a relaxation of* (**P**)*. The objective function value of* (**LR**) *is an upper bound for* (**P**)*.*

On addition to constraints (31), we develop the linear function $\overline{f}_j = a_j * B_j + b_j$ which is such that:

$$\sum_{i \in S} X_{ij} \le M_j^{coal} * \overline{f}_j \le M_j^{coal} * f_j \qquad \forall j \in C.$$

Corollary 2: *Let* (**LA**) *be the following linear approximation of problem* (**P**)*.*

$$Maximize: \qquad \mathcal{Z}^P(X,Y,B)$$

$$\text{Subject to:} \qquad (10),(12) \text{ to } (16)$$

$$\sum_{i \in S} X_{ij} \le M_j^{coal} * \overline{f}_j \quad \forall j \in C. \tag{32}$$

Constraints (32) are an inner approximation of (11). A solution to problem (**LA**) *is feasible for* (**P**)*. The objective function value of* (**LA**) *is a lower bound for* (**P**)*.*

Linearizing the objective function:

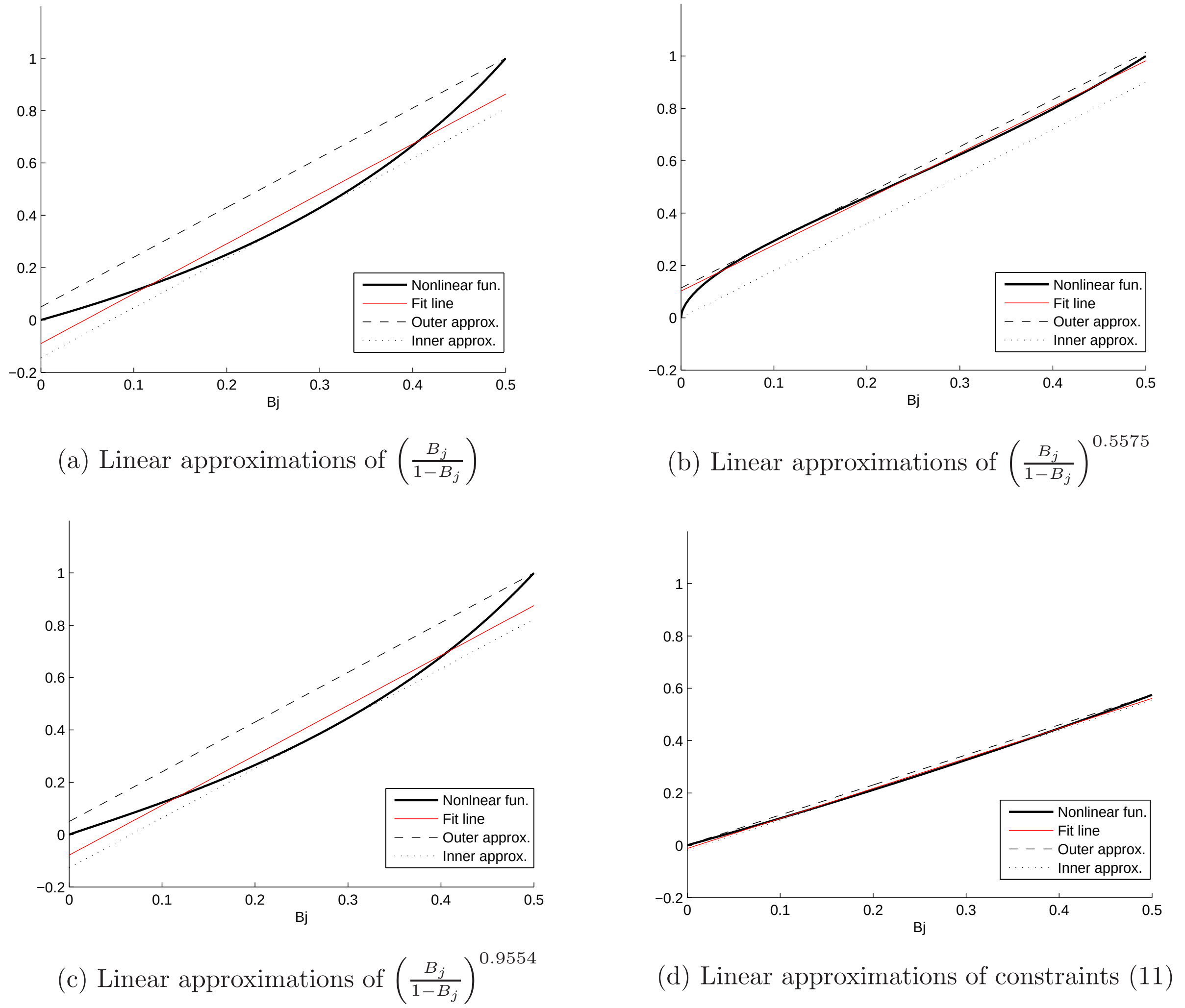
For $B_j \in [0, 0.5]$, the following nonlinear terms in the objective function, $f_j^1 = \left(\frac{B_j}{1-B_j}\right)$ is convex for $B_j \in [0, 0.5]$, and $f_j^2 = \left(\frac{B_j}{1-B_j}\right)^{9554}$ is convex for $B_j \in [0.04, 0.5]$ (see Propositions **6** and **7**). The nonlinear term $f_j^3 = \left(\frac{B_j}{1-B_j}\right)^{0.5575}$ is concave for $B_j \in [0, 0.22]$ and convex for $B_j \in (0.22, 0.5]$ (see Proposition **8**). For each of these three terms, we develop three linear approximations, one that overestimates the function (an outer approximation), one that underestimates the function (an inner approximation), and one that minimizes the squared error (fit line). Let $\overline{f}_j^{oi} = a_j^{oi} * B_j + b_j^{oi}$ be the outer approximation line of the i-th term (i = 1,2,3) for each $j \in C$. Let $\overline{f}_j^{ui} = a_j^{ui} * B_j + b_j^{ui}$ be the inner approximation line and $\overline{f}_j^{fi} = a_j^{fi} * B_j + b_j^{fi}$ the fit line. By substituting the non-linear terms in the objective function of (**P**), with the outer approximation lines we get the following, partial linearization of the objective function of (**P**).

$$Maximize: \sum_{i \in S}\sum_{j \in C} \overline{c}_{ij} X_{ij} - \sum_{j \in C} I_j^{cap}\left(a_j^{o1} B_j + b_j^{o1}\right) Y_j - \sum_{j \in C} I_j^h \left(a_j^{o2} B_j + b_j^{o2}\right)(1 - Y_j) -$$
$$- \sum_{j \in C} \left(I_j^s + I_j^{cd}\right)\left(a_j^{o3} B_j + b_j^{o3}\right)(1 - Y_j)$$

Rearranging the terms in the objective function we have:

$$Maximize: \sum_{j \in C}\left(\sum_{i \in S} \overline{c}_{ij} X_{ij} - \overline{a}_j^o B_j - \overline{b}_j^o B_j Y_j - \overline{d}_j^o Y_j - \overline{e}_j^o\right).$$

Where, $\overline{a}_j^o = I_j^h a_j^{o2} + \left(I_j^s + I_j^{cd}\right) a_j^{o3}$; $\overline{b}_j^o = I_j^{cap} a_j^{o1} - I_j^h a_j^{o2} - \left(I_j^s + I_j^{cd}\right) a_j^{o3}$; $\overline{d}_j^o = I_j^{cap} b_j^{o1} - I_j^h b_j^{o2} - \left(I_j^s + I_j^{cd}\right) b_j^{o3}$; and $\overline{e}_j^o = I_j^h b_j^{o2} + \left(I_j^s + I_j^{cd}\right) b_j^{o3}$. Figure 1 presents the liner approximations of the objective function and constraints (11).



(a) Linear approximations of $\left(\frac{B_j}{1-B_j}\right)$

(b) Linear approximations of $\left(\frac{B_j}{1-B_j}\right)^{0.5575}$

(c) Linear approximations of $\left(\frac{B_j}{1-B_j}\right)^{0.9554}$

(d) Linear approximations of constraints (11)

Figure 1 Linear Approximation Schemes

To get a fully linear objective function we introduce $Z_j = B_j Y_j$. Thus,

$$Z_j = \begin{cases} B_j & \text{if } Y_j = 1 \\ 0 & \text{if } Y_j = 0 \end{cases} \tag{33}$$

To represent this relationship using linear functions, we introduce additional variables. Let $\mathcal{Y}_j^1$ and $\mathcal{Y}_j^2$ be binary variables, and let w_j^1, w_j^2, w_j^3 be continuous variables in $[0,1]$. Let $(\mathbf{LA^u})$ be the following linear approximation of problem $(\mathbf{P})$. Equations (38) to (45) linearize the relationship between Z_j nd B_j.

$$Maximize: \mathcal{Z}^{LA^u} = \sum_{j\in C}\left(\sum_{i\in S}\overline{c}_{ij}X_{ij} - \overline{a}_j^u B_j - \overline{b}_j^u Z_j - \overline{d}_j^u Y_j - \overline{e}_j^u\right).$$

$$\sum_{j\in C} X_{ij} \leq s_i \qquad \forall i \in S \tag{34}$$

$$\sum_{i\in S} X_{ij} \leq M_j^{coal} * \overline{f}_j \qquad \forall j \in C \qquad (35)$$

$$B_j \leq 0.04 + M(1-Y_j) \qquad \forall j \in C \qquad (36)$$

$$B_j > 0.04(1-Y_j) \qquad \forall j \in C \qquad (37)$$

$$\mathcal{Y}_j^1 + \mathcal{Y}_j^2 = 1 \qquad \forall j \in C \qquad (38)$$

$$w_j^1 \leq \mathcal{Y}_j^1 \qquad \forall j \in C \qquad (39)$$

$$w_j^2 + w_j^3 = \mathcal{Y}_j^2 \qquad \forall j \in C \qquad (40)$$

$$0.04w_j^1 = Z_j \qquad \forall j \in C \qquad (41)$$

$$0.04w_j^1 + 0.04w_j^2 + 0.5w_j^3 = B_j \qquad \forall j \in C \qquad (42)$$

$$w_j^1, w_j^2, w_j^3, B_j \in [0,1] \qquad \forall j \in C \qquad (43)$$

$$\mathcal{Y}_j^1, \mathcal{Y}_j^2, Y_j \in \{0,1\} \qquad \forall j \in C \qquad (44)$$

$$X_{ij} \in R^+ \qquad \forall i \in S, j \in C \qquad (45)$$

Let ($\mathbf{LA^o}$) be the following liner approximation of problem ($\mathbf{P}$).

$$Maximize: \mathcal{Z}^{LA^o} = \sum_{j\in C}\left(\sum_{i\in S} \overline{c}_{ij} X_{ij} - \overline{a}_j^o B_j - \overline{b}_j^o Z_j - \overline{d}_j^o Y_j - \overline{e}_j^o\right).$$

Subject to: (34) to (45).

Let ($\mathbf{LA^f}$) be the following liner approximation of problem ($\mathbf{P}$).

$$Maximize: \mathcal{Z}^{LA^f} = \sum_{j\in C}\left(\sum_{i\in S} \overline{c}_{ij} X_{ij} - \overline{a}_j^f B_j - \overline{b}_j^f Z_j - \overline{d}_j^f Y_j - \overline{e}_j^f\right).$$

Subject to: (34) to (45).

Corollary 3: *A solution of problem (*$\mathbf{LA^u}$*) is feasible for problem (*$\mathbf{P}$*).*

Corollary 4: *A solution of problem (*$\mathbf{LA^o}$*) is feasible for problem (*$\mathbf{P}$*).*

Corollary 5: *A solution of problem (*$\mathbf{LA^f}$*) is feasible for problem (*$\mathbf{P}$*).*

Let ($\mathbf{LR^u}$) be the following liner approximation of problem ($\mathbf{P}$).

$$Maximize: \mathcal{Z}^{LR^u} = \sum_{j\in C}\left(\sum_{i\in S} \overline{c}_{ij} X_{ij} - \overline{a}_j^u B_j - \overline{b}_j^u Z_j - \overline{d}_j^u Y_j - \overline{e}_j^u\right).$$

Subject to: (34), (31), (36) to (45).

Proposition 3: Problem ($\mathbf{LR^u}$) is a relaxation of problem ($\mathbf{P}$), thus its objective function value is an upper bound for ($\mathbf{P}$).

Proof: Problem ($\mathbf{LR^u}$) is a relaxation of ($\mathbf{P}$) since the feasible region of ($\mathbf{LR^u}$) contains the feasible region of ($\mathbf{P}$). This is due to replacing constraints (35) with their corresponding outer approximation, constraints (31).

7. Numerical Analysis

We develop a case study in order to evaluate the impact of biomass co-firing on the production of renewable electricity. The case study is focused on the following 9 states located in the southeast: Alabama, Arkansas, Florida, Georgia, Louisiana, Mississippi, North Carolina, South Carolina and Tennessee. We focus on this region since it is rich with biomass. Numerical analysis is also used to evaluate the performance of the algorithms proposed.

7.1. Data Description

7.1.1. Biomass supply: Biomass availability data by state and county is extracted from the Knowledge Discovery Framework (KDF) database, an outcome of the US Billion Ton Study led by the Oak Ridge National Laboratory (KDF Accessed 12.10.2013). This database provides the amount of biomass available at the county level in the form of forest products, forest residues, agricultural products, agricultural residues, energy plants, etc. The database provides the amount of biomass available at different market prices for the 2012 to 2030 period.

From this data set we extracted and used data about forest products and residues. We focus the analysis on these products only, because, research has shown that these products are low in sulfur, as well as, chemicals such as, chlorine, potassium and nitrogen. These chemicals when burned cause corrosion and consequently impact the lifetime of burners. Thus, the chances are these will be the types of biomass used by coal plants.

7.1.2. Coal plants: The data about coal-fired power plant locations, nameplate capacities, types of coal used, utilization rates, and annual heat input rates, is collected from the US Energy Information Administration (2011). This database presents a total of 1,400 coal-fired power plants across the USA, with an overall nameplate capacity of 343,757 MW.

Table 1 summarizes the data about biomass available and coal plants in Southeast USA. The amounts of biomass listed represent the available biomass at $200/ton in 2014 based on KDF (Accessed 12.10.2013).

7.1.3. Truck transportation costs: In order to estimate costs for truck transportation of biomass, we use the data provided by Searcy et al. (2007). They provide two cost components which are the distance variable cost (DVC) and distance fixed cost (DFC). The distance variable cost includes the fuel and labor costs. The distance fixed cost includes the cost of loading and unloading a truck. These costs were provided for different types of biomass, such as, woodchips, straw and stover. We used the data provided for woodchips. The DVC of woodchips is estimated $0.112/(tons mile) and DFC is estimated $3.01/(tons). Woodchips are shipped using truck with a capacity of 40 tons. This data is used as follows in order to calculate $c_{ij}(\$/ton) = DFC + DVCd_{ij}$, where d_{ij} represents the distance between supplier i and plant j.

Table 1 **Distribution of biomass and coal plants in Southeast USA**

State	Biomass available (in tons)	Number of coal plants
AL	5,004,000	11
AR	4,505,800	4
FL	2,878,500	15
GA	6,892,500	14
LA	5,044,100	4
MS	5,772,200	5
NC	5,755,400	25
SC	3,666,300	16
TN	2,872,500	10

7.2. Experimental Results

The nonlinear model (**P**) is solved using GAMS/BONMIN solver. The linear approximation models are solved using Version 20141128 of AMPL an GUROBI 6.0.0. solver. The experiments are completed using a Dell personal computer with Intel(R) Core(TM) i5-4300U CPU @ 1.90GHz 2.50 GHz; and 8.00 GB of RAM. The following summarizes our experimental results.

7.2.1. Evaluating the quality of the upper and lower bounds: In order to test the performance of the upper and lower bounds proposed we randomly generate a number of problems. We tried solving model (**P**) using the overall dataset. However, BONMIN ran out of memory without finding a feasible solution due to the problem size. Thus, we solved model (**P**) using the data from Alabama. Next, we changed one problem parameter at a time and generated 8 different problems. For example, in Problems 1 and 2, biomass supply for each county in Alabama is generated randomly based on the intervals presented in the Table 2. For each problem we generated 5 random instances, and the results presented are the averages overall problem instances. The rest of problem parameters remain the same as the ones described in Section 7.1.

Table 2 **Summary of problem parameters**

Problem Nr.	Parameter	Random interval
1	Biomass supply (s_i): low	[0,10000]
2	Biomass supply (s_i): high	[0,40000]
3	Biomass cost (c_i^{bm}): low	[0,100]
4	Biomass cost (c_i^{bm}): high	[0,500]
5	Coal price (σ_j^p): low	[10,50]
6	Coal price (σ_j^p): high	[50,200]
7	Transportation cost (c_{ij}): low	[0,80]
8	Transportation cost (c_{ij}): high	[30,100]

Table 3 summarizes the results of the Lagrangean relaxation algorithm. The error gap $= \left(\frac{UB-LB}{LB}\right) * 100$. The quality of the upper bounds is very good. The maximum error bound is less than 4%.

Table 3 Upper bounds via Lagrangean relaxation

Problem Nr.	Error (%)	Lagr. Relaxation CPU (sec)	BONMIN CPU (sec)
1	0.78	671	368
2	0.01	49	410
3	1.28	813	578
4	3.69	911	128
5	0.00	70	170
6	0.00	33	96
7	0.00	79	439
8	0.36	109	502

We solved model (**P**) and its linear approximation model (**Q**) in order to evaluate the quality of the solutions provided by the linear approximation as a function of problem size ($|L|$). This analysis gave us an indication of what would be a good size for set L. We tried solving model (**P**) using the overall dataset. Initially, we solved model (**P**) using the data from Alabama. Next, we solved model (**Q**) several times using the same dataset. Each time we changed $|L|$. We focus on strategies for which the value of B_j is between 1% and 50% which are appropriate strategies for direct co-firing. As we increase $|L|$, we explore additional co-firing strategies within this range. For example, when $|L|=3$, the only strategies considered are $B_j = 0\%, 25\%$, and 50%. When $|L|=51$, then strategies considered are $B_j = 0\%, 1\%, 2\%, \ldots, 49\%, 50\%$.

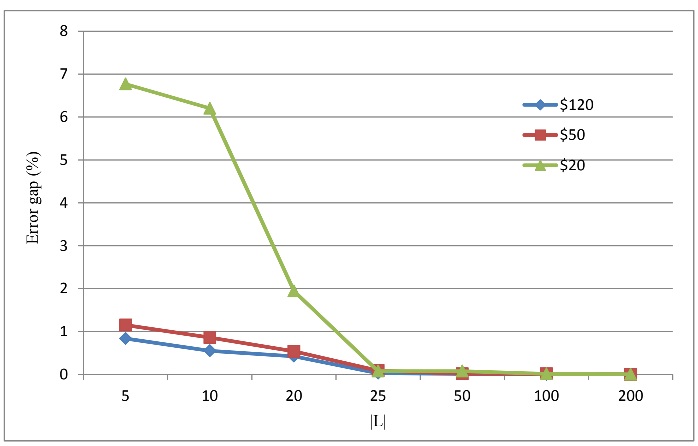


Figure 2 The error gap between $\mathcal{Z}^P$ and $\mathcal{Z}^Q$ as a function of $|L|$

Figure 2 presents the relationship between the size of $|L|$ and the relative error gap between the optimal solution to models (**Q**) and (**P**) when biomass market price is fixed at \$20/ton, \$50/ton and \$120/ton. The error gap is calculated as follows:

$$\text{Error gap} = \frac{(\mathcal{Z}^P - \mathcal{Z}^Q)}{\mathcal{Z}^P} * 100.$$

As expected, the relative error gap approaches zero as we increase the size of L. The results of this graph indicate that the error gap is smaller than 0.1 when the size of L is smaller than 25. In our numerical analysis we use $|L| = 200$ to ensure high quality solutions from the approximation.

Table 4 summarizes the running time of GUROBI when solving model (**Q**) and BONMIN when solving model (**P**). The running time of GUROBI increases only slightly due to the increase in problem size.

Table 4 Solution times in cpu seconds.

	Solution time	
$\|L\|$	**P**	**Q**
	297	
5		4.2
10		3.9
20		4.7
25		4.3
50		4.8
100		5.2
200		7.2

Table 5 summarizes the results from solving the linear approximation models presented in Sections 6.1 and 6.2. As indicated by Proposition **1** and Corollaries 2 to 5, by solving problems (**Q**), (**LA**), ($\mathbf{LA^o}$), ($\mathbf{LA^u}$) and ($\mathbf{LA^f}$) we generate feasible solutions for problem (**P**). We use these solutions to calculate lower bounds for (**P**). Based on these numerical results, the time it took to solve problem (**P**) using BONMIN is order of magnitude higher than the time required to solve problem (**Q**). The time required to solve problems ($\mathbf{LA^o}$), ($\mathbf{LA^u}$) and ($\mathbf{LA^f}$) is clearly the best, however, the quality of the corresponding solutions is poor.

7.2.2. Sensitivity analysis: Table 6 presents the total amount of biomass used by state as biomass price increases. Table 7 presents the total profits by state as biomass price increases. In order to generate these results we solved model (**Q**) for different values of biomass market price. Note that, not all of the available biomass is sold at the highest price, only the additional amount that becomes available at that price. Increasing the price positively impacts the amount of biomass that can be used for production of renewable energy. This is mainly because the amount of biomass made available to and used by power plants increases as plants are willing to pay a higher price.

Table 5 Comparing the performance of the lower bounds for problem (P).

Prob. Nr.	BONMIN CPU (sec)	(Q) Error (%)	(Q) CPU (sec)	(LA) Error (%)	(LA) CPU (sec)	(LA^u) Error (%)	(LA^u) CPU (sec)	(LA^o) Error (%)	(LA^o) CPU (sec)	(LA^f) Error (%)	(LA^f) CPU (sec)
1	368.00	0.00*	7.96	10.40	454.60	13.69	0.61	11.35	0.45	30.79	0.17
2	410.40	0.00	6.64	1.09	255.80	5.08	0.54	5.08	0.22	7.41	0.16
3	578.60	0.00	8.00	2.93	325.80	8.48	0.83	3.45	0.63	8.77	0.21
4	128.20	0.00	6.40	22.71	147.00	23.77	0.23	25.13	0.19	107.51	0.07
5	170.60	0.00	7.44	2.04	112.00	8.45	0.25	4.20	0.18	7.62	0.10
6	95.80	0.00	6.92	1.11	115.80	1.49	0.34	1.39	0.28	2.80	0.17
7	439.00	0.09*	8.23	1.05	220.00	2.12	0.29	1.58	0.14	3.91	0.14
8	502.33	0.00*	9.00	3.14	295.00	6.70	0.22	5.36	0.17	12.58	0.08

*The quality of solutions from solving model (Q) are slightly better than solutions found from BONMIN, although BONMIN reports 0% error gap for these solutions.

Table 6 The total biomass used at different levels of biomass market price

Biomass price (in $/ton)	Total biomass used (in mill tons) AL	AR	FL	GA	LA	MS	NC	SC	TN
20	1.40	0.04	1.15	1.75	0.44	1.62	1.81	0.73	1.45
40	4.27	0.85	4.40	5.22	1.16	3.32	4.87	3.30	3.97
60	4.48	0.90	4.69	5.43	1.17	3.36	5.29	3.29	4.10
80	4.60	0.90	4.73	5.51	1.17	3.36	5.35	3.33	4.13
100	4.82	0.90	4.73	5.51	1.17	3.36	5.35	3.33	4.13
140	4.82	0.90	4.73	5.51	1.17	3.36	5.35	3.33	4.13
200	4.82	0.90	4.73	5.51	1.17	3.36	5.35	3.33	4.13

The results of Figures 6 and 7 indicate that, the amount of biomass used depends on the number of coal plants, rather than biomass availability within the state. For example, North Carolina, South Carolina, Florida, Georgia and Tennessee use most of the biomass available in the region. This is because the number of coal plants in these states varies between 10 and 25. The number of coal plants in the remaining states in Southeast is smaller (see Table 1). Therefore, these states become biomass suppliers to states that have more coal plants.

Let consider the case of Florida. Biomass availability in Florida (at the highest market price of $200/ton) is close to 2.88 million tons. However, the amount of biomass used in Florida, based on our numerical results, is 4.73 million tons at a market price of $80/ton. The corresponding system wide profits are $213 million. In this case, although biomass is produced in other states within the region, the benefits of the PTC will be collected by Florida. Similarly, Tennessee produces only 2.87 million tons of biomass. Based on our model, Tennessee would use up to 4.13 million tons of biomass and generate up to $152 million in profits mainly due to PTC. On the other side, states such as Arkansas and Louisiana that are rich in biomass (over 5 million tons of biomass available each) would make a small profit of $0.9 million and $1.17 million correspondingly.

Table 7 Total profits at different levels of biomass market price

Biomass price	Total profits (in mill $)								
in ($/ton)	AL	AR	FL	GA	LA	MS	NC	SC	TN
20	172.93	0.98	65.94	100.28	12.68	67.64	104.58	45.69	67.36
40	376.20	8.90	206.48	247.77	22.57	108.54	235.01	160.27	150.62
60	378.73	9.16	212.80	251.60	23.30	108.57	247.54	155.99	152.65
80	378.67	9.22	213.21	252.27	23.54	108.53	247.07	156.86	152.41
100	378.81	9.16	213.21	252.27	23.51	108.85	247.21	156.86	151.90
140	378.74	9.23	213.08	252.41	23.50	108.90	247.32	156.49	152.12
200	379.35	9.16	213.06	252.42	23.81	108.04	247.48	156.33	152.12

The results of Table 7 indicate that biomass usage and total profits remain the same as the market price increases beyond $80/ton. Using the additional biomass which becomes available at the higher market price decreases profits. This is because the additional tax savings are smaller than the additional purchase, transportation and investment costs necessary to use the additional biomass.

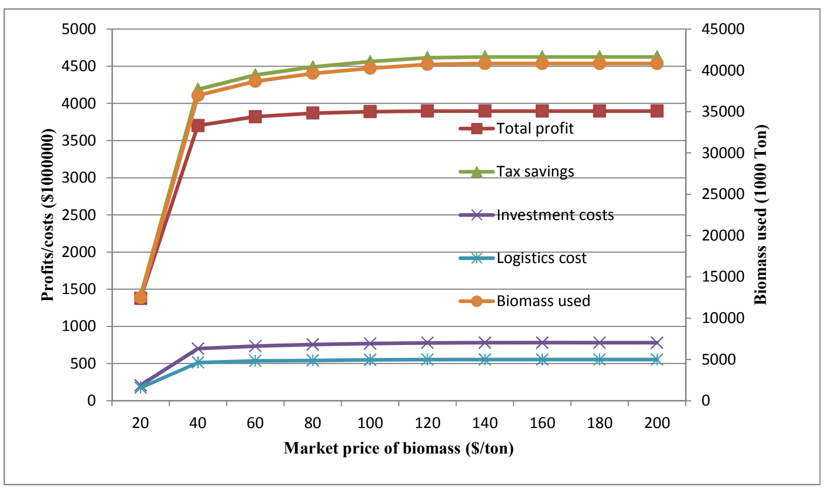
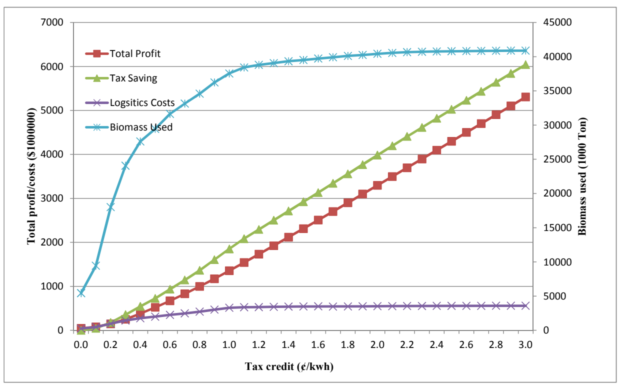


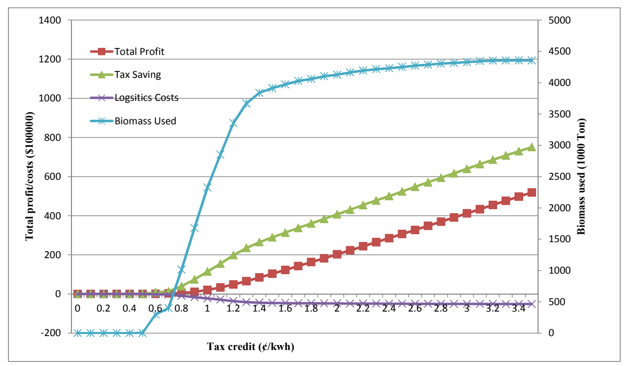
Figure 3 **Analyzing the impact of biomass market price on average profits, costs and biomass usage in Southeast USA**

Figure 3 presents the relationship that exists between biomass market price and total profits, tax savings, biomass used, and investment and logistics costs. The PTC is fixed at 1.1 cents per kilowatt-hour. As the market price of biomass increases from $20/ton to $80/ton, the amount of biomass available and overall system profits increase. The rate of increase of profits is higher when the market price increases from \$20 to \$40/ton. The amount of biomass used and corresponding profits do not change at market prices higher that \$80/ton since the additional tax savings are smaller than the additional purchase, transportation and investment costs.

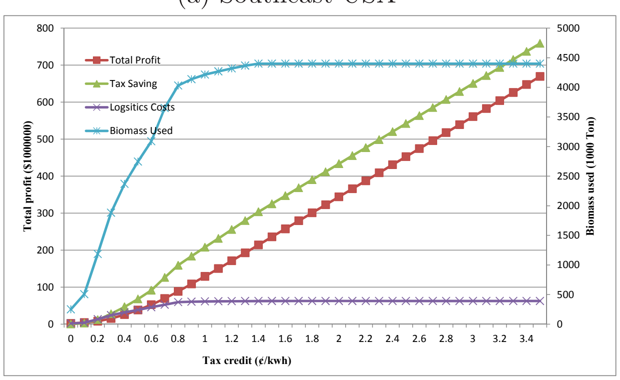
Figure 4 depicts the relationship between PTC and total profits, tax savings, biomass used, and investment and logistics costs in Southeast US as well as, in Arkansas, Mississippi and South Carolina. The market price of biomass here is fixed at \$100/ton. Results indicate that, when PTC is equal to zero, the average benefits from co-firing biomass - although small- are positive. Plants find co-firing to be beneficial when PTC is zero, with the exception of plants located in Arkansas and Louisiana. In Arkansas, coal plants would use biomass for co-fire when PTC is greater than 0.7 cents per kilowatt-hour (Figure 4(b)); and in Louisiana when PTC is greater than 0.2 cents per kilowatt-hour.



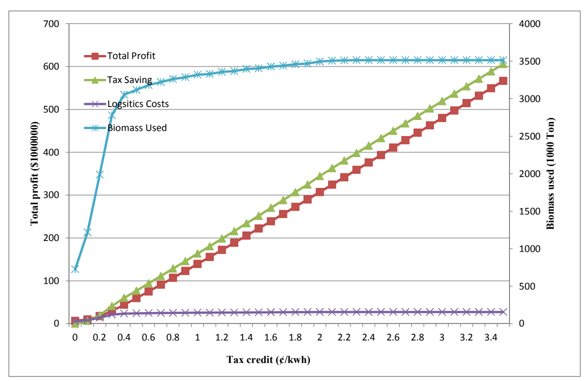
(a) Southeast USA



(b) Arkansas



(c) Mississippi



(d) South Carolina

Figure 4 The relationship between total profits and tax credits

The results of Figure 4(a) indicate that an increase of PTC from 0 to 1 cent per kilowatt-hour has a dramatic impact on biomass usage in Southeast. The amount of biomass used increases 4.8 times. Increasing the PTC from 1 to 2 cents per kilowatt-hour increases biomass usage by 6%; and increasing PTC from 2 to 3 cents per kilowatt-hour increases biomass usage by 0.5%. These results indicate that the impact of increasing PTC beyond 2 cents per kilowatt-hour on the total amount of renewable electricity produced in Southeast is only marginal. The corresponding increase in total profits is just due to the higher PTC, and it is not due to increase use of biomass used.

8. Summary and Conclusions

Co-firing biomass in coal-fired power plants is a strategy that leads to reduced greenhouse gas emissions. This paper presents a mathematical model to evaluate the impact of biomass co-firing in generating renewable electricity. The model captures the additional biomass purchasing and transportation costs, plant investment costs, savings due to PTC, and savings from reducing the amount of coal purchased. The model also captures the loss in process efficiencies due to using biomass, a product which has lower heating value as compared to coal. The model proposed is a MINLP, thus, we present a linear approximation which is easier to solve. We use numerical analysis to evaluate the quality of solutions from the linear approximation model.

We develop a case study using data from nine states located in the southeast region of the US. The data source used are the Knowledge Discovery Framework KDF (Accessed 12.10.2013) and the US Energy Information Administration (2011). These databases provide information about the available amount of biomass for production of renewable energy by county and state, at different market prices, during the period 2012 to 2020. The databases also provide detailed information about the coal-fired power plants in the US. We used this data and conducted an extensive number of experiments. The following summarizes our main observations:

Observation 1: *Tax credits do have an impact in increasing the production of renewable energy.* The results of Figure 4 indicate that increasing the PTC impacts greatly the production of renewable electricity. Our numerical results indicate that increasing PTC beyond 2 cents/kilowhat hour has only marginal impacts in increasing renewable energy generation.

Observation 2: *Tax credit should not be "one size fits all"*. Instead, tax credits could be a function of the amount of renewable electricity produced, or plant capacity.

The results of Figure 3 indicate that the amount of biomass used increases only slightly with the increase of biomass market price beyond $80/ton given that PTC is fixed at 1.1 cents per kilowatt-hour. Since biomass is a bulk product with low energy density and widely dispersed geographically, collection and transportation costs are high. For every 1% increase in biomass usage, the corresponding increase of transportation and collection costs is higher. In order to encourage the production of renewable energy, it makes sense to design a PTC which is a function of the amount of biomass used, and consequently, a function of the amount of renewable energy produced.

Production tax credits based on coal plant capacity are being currently implemented in European countries (KPM 2012, IEA 2015). Typically, the tax credit (such as, the "feed-in tariff" implemented in Austria) is higher for smaller sized plants. Higher credits allow smaller plants to overcome the burdens of implementing biomass co-firing.

Observation 3: *There is a need for comprehensive tax credit schemes to encourage renewable electricity production and reduce GHG emissions.* Biomass distribution in the US differs by region, and it does not match the distribution of coal-fired power plants (Figure 1). Therefore, in our case study, some states of Southeast became biomass suppliers to other states that do currently have a larger number of power plants. Consequently, states that have the resources to transform biomass to renewable electricity rip the gains from PTC. Recall that, one of the main reasons of producing renewable energy is to reduce GHG emissions due to burning of coal. Clearly, when biomass is transported over state borders, the transportation distances and corresponding GHG emissions do increase. Further increases of PTC would allow power plants to remain profitable even if biomass is delivered from suppliers located far away. Thus, decisions related to PTC size and scheme should be mindful of of the impacts of PTC to GHG emissions due to co-firing and transportation in the supply chain.

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Appendix A

Theorem 1: *In an optimal solution* $X^* = \{X_1^*, X_2^*, \ldots, X_l^*\}$ to the single plant problem, at most one of the suppliers' is used partially. That means, $X_i^* = s_i$, $\forall i \in S^*/k$; $X_k^* = \gamma$; and $X_i^* = 0$ for $i \notin S^*$. Where, S^* is the set of suppliers selected in the optimal solution.

Proof:

Let's assume that the suppliers within set S are sorted in an increasing order of the size of variable unit costs $(c_i + c_i^{bm})$. Thus, $(c_1 + c_1^{bm}) \leq \ldots \leq (c_i + c_i^{bm}) \leq \ldots \leq (c_l + c_l^{bm})$, where, $l = |S|$. Let the unit profit be $\overline{c}_i = (\sigma^p + \sigma^t - c_i - c_i^{bm})$. Thus, the unit profits are decreasing, and $\overline{c}_1 \geq \overline{c}_2, \ldots, \geq \overline{c}_l$.

Let's divide the interval $[0, 0.5]$ into k smaller subintervals as follows $[0, B_1], [B_1, B_2], \ldots[B_{k-1}, 0.5]$. The breakpoint B_1 represents what percentage of coal that could be displaced if all the biomass available at supplier 1 is being used by the plant. Next, B_2 represents the percentage of coal that can be displaced if biomass supply available at suppliers 1 and 2 was used; and so on. Without loss of generality, we assume that $l \geq k$.

Increasing the value of B impacts the amount of biomass used at the plant, thus, the right hand-side values of constraints (11) increase with B. Let $M_1, M_2, \ldots, M_k$ represents the amount of biomass required at $B_1, B_2, \ldots, B_k$. Indeed, $M_1 = s_1$ since we assumed that supplier 1 is being used to meet the demand for biomass as B increases from 0 to B_1; $M_2 = s_1 + s_2$; and so on.

For fixed values of B, we can also calculate corresponding investment costs using equations (4) and (5). Let $I_1, I_2, \ldots I_k$ be the investment costs at the corresponding breakpoints $B_1, B_2, \ldots, B_k$.

Let $\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_k$ denote the total cost for $B \in \{B_1, B_2, \ldots, B_k\}$. Therefore, $\mathcal{Z}_l = \sum_{i=1}^{l} \overline{c}_i X_i - I_l$ for $l = 1, \ldots, k$.

Theorem 1 implies that in an optimal solution the total amount of biomass used is equal to: $M^* = \sum_{i=1}^{j} X_i^* = \sum_{i=1}^{j-1} s_i + \gamma$, and $\gamma \leq s_j, j \leq k$. We prove this by contradiction. Let's assume that $X^* = \{X_1^*, X_2^*, \ldots, X_f^*\}$ is an optimal solution to the single plant problem. This solution is such that more than one of the suppliers is used partially. That means: $X_1^* < s_1, X_2^* < s_2, \ldots, X_f^* < s_f$, and $\sum_{i=1}^{f} X_i^* = M^*$. Let I^* denote the corresponding investment costs. Since X^* is the optimal solution to this problem, then $\mathcal{Z}(X^*) \geq \mathcal{Z}(X)$ for any feasible X. Let $X = \{X_1, X_2, \ldots, X_j\}$ be a feasible solution such that: $X_1 = s_1, X_2 = s_2, \ldots, X_{j-1} = s_{j-1}, X_j = \gamma$; and $\sum_{i=1}^{j} X_i = M^*$. Since the total amount of biomass used in both solutions is the same, M^*, the corresponding investment costs are the same. In solution X all the suppliers are completely used, thus, $j < f$.

$$\mathcal{Z}(X^*) - \mathcal{Z}(X) = \left(\sum_{i=1}^{f} \overline{c}_i X_i^* - I^*\right) - \left(\sum_{i=1}^{j} \overline{c}_i X_i - I^*\right) = \sum_{i=1}^{f} \overline{c}_i X_i^* - \sum_{i=1}^{j-1} \overline{c}_i s_i - \overline{c}_j \gamma =$$

$$= \sum_{i=1}^{f} \overline{c}_i X_i^* - \sum_{i=1}^{j-1} \overline{c}_i X_i^* - \sum_{i=1}^{j-1} \overline{c}_i (s_i - X_i^*) - \overline{c}_j \gamma$$

$$= \sum_{i=j}^{f} \overline{c}_i X_i^* - \sum_{i=1}^{j-1} \overline{c}_i (s_i - X_i^*) - \overline{c}_j \gamma.$$

Since $\sum_{i=1}^{f} X_i^* = \sum_{i=1}^{j} X_i$, this is true: $\sum_{i=j}^{f} X_i^* = \sum_{i=1}^{j-1}(s_i - X_i^*) + \gamma$. Since $\overline{c}_1 \geq \overline{c}_2, \ldots \geq \overline{c}_j \ldots \geq \overline{c}_f$, the following holds true:

$$\mathcal{Z}(X^*) - \mathcal{Z}(X) = \sum_{i=j}^{f} \overline{c}_i X_i^* - \sum_{i=1}^{j-1} \overline{c}_i (s_i - X_i^*) - \overline{c}_j \gamma \leq \overline{c}_j \sum_{i=j}^{f} X_i^* - \overline{c}_j \sum_{i=1}^{j-1} (s_i - X_i^*) - \overline{c}_j \gamma = 0.$$

Therefore, $\mathcal{Z}(X^*) \leq \mathcal{Z}(X)$. This contradicts the assumption that $\mathcal{Z}(X^*)$ is an optimal solution for the single plant problem. Therefore, in an optimal solution at most one of the suppliers is used partially. ■

Theorem 2. *The special case of problem* (**P**) *-when there is a single plant in the supply chain- can be solved to optimality via an* $O(|S| log(|S|))$ *algorithm.*

Proof:

Let's assume that the suppliers within set S are sorted in an increasing order of the size of variable unit costs $(c_i + c_i^{bm})$. Let's divide the interval $[0, 0.5]$ into k smaller subintervals $[0, B_1], [B_1, B_2], \ldots [B_{k-1}, 0.5]$ in a similar way as explained in Theorem 1. Without loss of generality, we assume that $\overline{c}_i > 0$ for $i = 1, \ldots, k$. We analyze the characteristics of the solution to (**P**) for $B \in [0, 0.04]$, $B \in [0.04, 0.221)$, and $B \in (0.221, 0.5]$.

Case 1: $B^* \in [0, 0.04]$.

Let (X^*, B^*, Y^*) denote a solution to the single plant case of problem (**P**). Since $B^* \leq 0.04$, then $Y^* = 1$. Let's assume that suppliers $1, \ldots, i$ were selected in the optimal solution. Thus, $B^* \in [B_{i-1}, B_i]$. Based on Theorem 1, and since constraints (22) are binding (Proposition **9**), the optimal objective function value is

$$\mathcal{Z}(B^*) = \sum_{j=1}^{i-1} \overline{c}_j s_j + \overline{c}_i \left(\frac{M^{coal} \rho^b B^*}{(1 - \alpha B^*)(\rho^b - 0.0044 (B^*)^2 - 0.0055)} - \sum_{j=1}^{i-1} s_j \right) - I^{cap} \left(\frac{B^*}{1 - B^*} \right).$$

Let function

$$f_i^1(B) = \frac{(\overline{c}_i M^{coal} \rho^b) \, B}{(1 - \alpha B)(\rho^b - 0.0044 B^2 - 0.0055)}$$

and

$$f_i^2(B) = I^{cap} \left(\frac{B}{1 - B} \right).$$

Thus,

$$\mathcal{Z}(B^*) = \sum_{j=1}^{i-1} \overline{c}_j s_j - \overline{c}_i \sum_{j=1}^{i-1} s_j + f_i^1(B^*) - f_i^2(B^*) = \sum_{j=1}^{i-1} (\overline{c}_j - \overline{c}_i) s_j + f_i^1(B^*) - f_i^2(B^*).$$

Functions $f_i^1(B)$ and $f_i^2(B)$ are continuous functions of B for $B \in [B_{i-1}, B_i]$. Thus, the objective function $\mathcal{Z}(B)$ is also continuous on B (Hazewinkel 2001). Since $\mathcal{Z}(B)$ is the difference of two convex functions, we cannot conclude that it is concave, or, that it is convex. Let $\delta_i^1(B)$ denote the slope of function $f_i^1(B)$; and $\delta_i^2(B)$ denote the slope of function $f_i^2(B)$. Since both functions are increasing, $\delta_i^1(B) > 0$ and $\delta_i^2(B) > 0$. The following cases could be encounter for $B \in [B_{i-1}, B_i]$.

(a) $\delta_i^1(B) > \delta_i^2(B)$: since the slope of $\mathcal{Z}(B) > 0$, $\mathcal{Z}(B)$ is increasing, thus, $B^* = B_i$.

(b) $\delta_i^1(B) < \delta_i^2(B)$: since the slope of $\mathcal{Z}(B) < 0$, $\mathcal{Z}(B)$ is decreasing, thus, $B^* = B_{i-1}$.

(c) $\delta_i^1(\overline{B}) = \delta_i^2(\overline{B})$ for some $\overline{B} \in [B_{i-1}, B_i]$: Since both $f_i^1(B)$ and $f_i^2(B)$ are strictly increasing in B, the slope of these lines could be equal in at most two points. Thus, $\mathcal{Z}(B)$ could be quasi-convex, or quasi-concave, or have one minimum and one maximum in $B \in [B_{i-1}, B_i]$. If $\mathcal{Z}(B)$ is quasi-convex, $B^* = \arg\max\{\mathcal{Z}(B_{i-1}), \mathcal{Z}(B_i)\}$. If $\mathcal{Z}(B)$ is quasi-concave, or have one minimum and one maximum, then, $B^* \in (B_{i-1}, B_i)$.

Case 2: $B^* \in [0.04, 0.221)$.

The optimal objective function value is

$$\mathcal{Z}_i(B^*) = \sum_{j=1}^{i-1} \overline{c}_j s_j + \overline{c}_i \left(\frac{M^{coal} \rho^b B^*}{(1-\alpha B^*)(\rho^b - 0.0044 (B^*)^2 - 0.0055)} - \sum_{j=1}^{i-1} s_j \right)$$
$$- I^h \left(\frac{B^*}{1-B^*} \right)^{0.9554} - \left(I^s + I^{cd} \right) \left(\frac{B^*}{1-B^*} \right)^{0.5575}.$$

Let function

$$f_i^1(B) = \frac{(\overline{c}_i M^{coal} \rho^b) B}{(1-\alpha B)(\rho^b - 0.0044 B^2 - 0.0055)} - \left(I^s + I^{cd} \right) \left(\frac{B}{1-B} \right)^{0.5575}$$

and

$$f_i^2(B) = I^h \left(\frac{B}{1-B} \right)^{0.9554}.$$

Thus,

$$\mathcal{Z}_i(B^*) = \sum_{j=1}^{i-1} (\overline{c}_j - \overline{c}_i) s_j + f_i^1(B^*) - f_i^2(B^*)$$

Functions $f_i^1(B)$ and $f_i^2(B)$ are continuous functions of B. Thus, the objective function $\mathcal{Z}(B)$ is also continuous on B (Hazewinkel 2001). Function $f_i^1(B)$ is convex since it is the difference of a convex and a concave function. However, we cannot say the same for function $\mathcal{Z}(B)$ since it is the difference of two convex function. Let $\delta_i^1(B)$ denote the slope of function $f_i^1(B)$; and $\delta_i^2(B)$ denote the slope of function $f_i^2(B)$. Since both functions are increasing, $\delta_i^1(B) > 0$ and $\delta_i^2(B) > 0$. The following cases could be encounter for $B \in [B_{i-1}, B_i]$.

(a) $\delta_i^1(B) > \delta_i^2(B)$: since the slope of $\mathcal{Z}(B) > 0$, $\mathcal{Z}(B)$ is increasing, thus, $B^* = B_i$.

(b) $\delta_i^1(B) < \delta_i^2(B)$: since the slope of $\mathcal{Z}(B) < 0$, $\mathcal{Z}(B)$ is decreasing, thus, $B^* = B_{i-1}$.

(c) $\delta_i^1(\overline{B}) = \delta_i^2(\overline{B})$ for some $\overline{B} \in [B_{i-1}, B_i]$: If $\mathcal{Z}(B)$ is quasi-convex, $B^* = \arg\max\{\mathcal{Z}(B_{i-1}), \mathcal{Z}(B_i)\}$. If $\mathcal{Z}(B)$ is quasi-concave, or have one minimum and one maximum, then, $B^* \in (B_{i-1}, B_i)$.

Case 3: $B \in (0.221, 0.5]$.

The optimal objective function value is

$$\mathcal{Z}_i(B^*) = \sum_{j=1}^{i-1} \overline{c}_j s_j + \overline{c}_i \left(\frac{M^{coal} \rho^b B^*}{(1-\alpha B^*)(\rho^b - 0.0044(B^*)^2 - 0.0055)} - \sum_{j=1}^{i-1} s_j \right)$$

$$-I^h \left(\frac{B^*}{1-B^*} \right)^{0.9554} - \left(I^s + I^{cd} \right) \left(\frac{B^*}{1-B^*} \right)^{0.5575}.$$

Let function

$$f_i^1(B) = \frac{(\overline{c}_i M^{coal} \rho^b)\, B}{(1-\alpha B)(\rho^b - 0.0044B^2 - 0.0055)}$$

and

$$f_i^2(B) = I^h \left(\frac{B}{1-B} \right)^{0.9554} + \left(I^s + I^{cd} \right) \left(\frac{B}{1-B} \right)^{0.5575}.$$

Thus,

$$\mathcal{Z}_i(B^*) = \sum_{j=1}^{i-1} (\overline{c}_j - \overline{c}_i) s_j + f_i^1(B^*) - f_i^2(B^*)$$

Functions $f_i^1(B)$ and $f_i^2(B)$ are continuous functions of B. Thus, the objective function $\mathcal{Z}(B)$ is also continuous on B (Hazewinkel 2001). Function $f_i^2(B)$ is convex since it is the sum of two convex functions. We cannot conclude whether function $\mathcal{Z}(B)$ is concave or convex since it is the difference of two convex functions. Let $\delta_i^1(B)$ denote the slope of function $f_i^1(B)$; and $\delta_i^2(B)$ denote the slope of function $f_i^2(B)$. Since both functions are increasing, $\delta_i^1(B) > 0$ and $\delta_i^2(B) > 0$. The following cases could be encounter while solving $\mathcal{Z}_i(B)$.

(a) $\delta_i^1(B) > \delta_i^2(B)$: since the slope of $\mathcal{Z}(B) > 0$, $\mathcal{Z}(B)$ is increasing, thus, $B^* = B_i$.

(b) $\delta_i^1(B) < \delta_i^2(B)$: since the slope of $\mathcal{Z}(B) < 0$, $\mathcal{Z}(B)$ is decreasing, thus, $B^* = B_{i-1}$.

(c) $\delta_i^1(\overline{B}) = \delta_i^2(\overline{B})$ for some $\overline{B} \in [B_{i-1}, B_i]$: If $\mathcal{Z}(B)$ is quasi-convex, $B^* = \arg\max\{\mathcal{Z}(B_{i-1}), \mathcal{Z}(B_i)\}$. If $\mathcal{Z}(B)$ is quasi-concave, or have one minimum and one maximum, then, $B^* \in (B_{i-1}, B_i)$.

Implications of the results from Cases 1-3.

Based on Theorem 1, at most one supplier will be used partially in an optimal solution. Additionally, we did sort the suppliers in a decreasing order of their unit profit $\overline{c}$. That means, if suppliers $1, \ldots, j$ are selected in an optimal solution, supplier j (the last supplier selected) is the only one

Table 8 A polynomial time algorithm for the single plant problem.

Step 1:	Initialize: $S^* = \emptyset$; $B^* = 0$; $M_i = B_i = I_i = 0$ for $i = 1, \ldots, k$, $k \leq \|S\|$. $l = 1$
Step 2:	Let $B^1 = 0.04, B^2 = 0.221, B^3 = 0.5$.
Step 3:	**Sort** $s_i \in S$ in an ascending order of unit profit $\overline{c}_i$.
Step 4:	**For** $i = 1, \ldots, k$:
	Let $M_i = M_{i-1} + s_i$
	Calculate B_i using equation (2)
	Calculate $\mathcal{Z}(B_i)$
	End For i
Step 5:	**For** $p = 1, \ldots 3$:
	For $i = l, \ldots k$:
	If $\mathcal{Z}(B_i) > 0$
	If $\mathcal{Z}(B_i) > \mathcal{Z}(B_{i-1})$ and $B_i \leq B^p$ then
	$S^* = S^* \bigcup i$
	$\phi = i$
	Else
	Find $B^{p*} \in [B_{i-1}, B_i]$ using Golden Search Algorithm
	If $B^{p*} > B_{i-1}$, then $S^* = S^* \bigcup i$, $\phi = i$
	End If
	End If
	End For i
	$l = \phi$
	End For p
	$B^* = \arg\max\{\mathcal{Z}(B^{1*}), \mathcal{Z}(B^{2*}), \mathcal{Z}(B^{3*})\}$
Step 6:	Report the optimal solution:
	B^*
	For $i = 1, \ldots, \|S^*\| - 1$, let $X_i^* = s_i$.
	Let $j = \|S^*\|$. Calculate X_j^*.
	If $B^* \leq 0.04$ then $Y^* = 1$, **Else**, $Y^* = 0$.

that could have been used partially. To find an optimal solution to the problem we need to compare the objective function values for each $B \in \{B_1, \ldots, B_l\}$.

■

Proposition 1: *A feasible solution to problem (**Q**) is feasible to the non-linear problem (**P**); and the objective function value of (**Q**) is a lower bound for problem (**P**).*

Proof: This is true due to the way we constructed model (**Q**). We built model (**Q**) by discretizing the continuous variable B_j. L is the set of the co-firing strategies that are investigated, thus, it includes some, but clearly not all the potential values that B_j could take.

Let $\Xi(\mathbf{Q})$ be the set of solutions to problem (**Q**), and $\Xi(\mathbf{P})$ be the set of solutions to problem (**P**). Let $(X_{ij}^Q, Y_{lj}^Q) \in \Xi(Q)$ be a feasible solution to problem (**Q**). We can use this solution to derive a feasible solution for problem (**P**) in the following way.

$$X_{ij}^P = X_{ij}^Q \text{ for } i \in I, j \in C \tag{46}$$

$$B_j^P = \sum_{l \in L} L_l * Y_{lj}^Q for j \in C \tag{47}$$

$$Y_j^P = \begin{cases} 1 & \text{if } \sum_{l \in L} L_l * Y_{lj}^Q \leq 0.04 \\ 0 & \text{otherwise.} \end{cases} \tag{48}$$

Since (X_{ij}^Q, Y_{lj}^Q), a feasible solution to (**Q**), can be used to derive a feasible solution for (**P**); it can easily be verified that the corresponding objective function value of (**Q**) is a lower bound for (**P**). ■

Theorem 3: *A*s $|L|$ approaches infinity, an optimal solution to (**Q**) is optimal to (**P**) with probability 1.

Proof: An optimal solution to problem (**Q**) could be transformed to a feasible solution for (**P**) using equations (46) to (48). Let, $\mathcal{Z}^{Q*}$ be the corresponding objective function value of (**Q**), and $\mathcal{Z}^P$ be the corresponding objective function value of (**P**). By construction, these two objective function values are equal. Next, we show that, under certain conditions, an optimal solution to (**P**) can be transformed to a feasible solution to (**Q**). Let $(X_{ij}^{P*}, B_j^{P*}, Y_j^{P*})$ be the optimal solution to problem (**P**).

CASE 1: $|L|$ is finite.

If $B_j^{P*} \in L$, then, we can derive a feasible solution for (**Q**) using the following equations.

$$X_{ij}^Q = X_{ij}^{P*} \text{ for } i \in I, j \in C \tag{49}$$

$$Y_{lj}^Q = \begin{cases} 1 & \text{if } B_j^{P*} = L_l \\ 0 & \text{otherwise} \end{cases} \quad \text{for } j \in C. \tag{50}$$

However, given that B_j^{P*} is a continuous variable, and L is a finite set, the probability that B_j^{P*} is represented in L is equal to zero.

If $B_j^{P*} \notin L$, then, we cannot derive an solution to model (**Q**) using a solution to (**P**).

CASE 2: $|L|$ is infinite.

In this case, $\lim_{|L| \to \inf} P(B_j^{P*} \in L) = 1$. This implies that, given an optimal solution to (**P**) we can derive a feasible solution to (**Q**) using equations (49) and (50). Let, $\mathcal{Z}^{P*}$ be the corresponding objective function value of (**P**), and $\mathcal{Z}^Q$ be the corresponding objective function value of (**Q**). These two objective function values are equal. By Proposition **1** and optimality theory, we have: $\mathcal{Z}^P \leq \mathcal{Z}^{P*} = \mathcal{Z}^Q \leq \mathcal{Z}^{Q*}$ and $\mathcal{Z}^{P*} \geq \mathcal{Z}^Q$. This implies that $\mathcal{Z}^{P*} = \mathcal{Z}^{Q*}$.

To summarize, as $|L| \to \inf$ the optimal solutions to models (**Q**) and (**P**) are equal with probability 1. ■

Proposition 2:

$$B_j \leq \frac{1}{M_j^{coal}} \sum_{i \in S} X_{ij} \leq f_j \leq (B_j + \overline{a}_j) \qquad \forall j \in C.$$

Where, $\overline{a}_j = \frac{0.5\left(\rho_j^b - (1-0.5\alpha_j)(\rho_j^b - 0.0066)\right)}{(1-0.5\alpha_j)(\rho_j^b - 0.0066)}$.

Proof: Function f_j has the following properties: $f_j = 0$ if $B_j = 0$ and $f_j > B_j$ for $B_j > 0$. This is due to the way we construct f_j. If B_j represents the percentage of coal being substituted in plant j (by mass), then, f_j transforms this percentage into an equivalent percentage of biomass needed to enable this substitution. Thus, the linear function $\overline{f}_j = B_j$ is an inner approximation of f_j. Indeed, $\overline{f}_j$ underestimates f_j for $B_j \in (0, 0.5]$.

Function f_j is an increasing function of B_j. Thus the difference $f_j - B_j$ reaches its maximum when $B_j = 0.5$. This maximum difference is $\overline{\alpha}_j$. To summarize,

$$f_j = 0 \quad \text{if} \quad B_j = 0$$

$$f_j = B_j + \overline{\alpha}_j \quad \text{if} \quad B_j = 0.5$$

$$B_j < f_j < B_j + \overline{\alpha}_j \quad \text{if} \quad 0 < B_j < 0.5.$$

■

Proposition 4: Function $f(B_j) = \frac{B_j}{(1-B_j)}$ is increasing in B_j, for $0 \leq B_j \leq 1$.

Proof: To proof this we show that for any $\epsilon > 0$, such that, $B_j + \epsilon < 1$, the following holds true $f(B_j + \epsilon) - f(B_j) \geq 0$.
Therefore,

$$f(B_j+\epsilon) - f(B_j) = \frac{B_j + \epsilon}{(1-(B_j+\epsilon))} - \frac{B_j}{(1-B_j)} = \frac{(B_j+\epsilon)(1-B_j) - B_j(1-(B_j+\epsilon))}{(1-(B_j+\epsilon))(1-B_j)} =$$

$$= \frac{B_j - B_j^2 + \epsilon - \epsilon B_j - B_j + B_j^2 + \epsilon B_j}{(1-(B_j+\epsilon))(1-B_j)} = \frac{\epsilon}{(1-(B_j+\epsilon))(1-B_j)} > 0.$$

■

Proposition 5: Functions $f(B_j) = \left(\frac{B_j}{1-B_j)}\right)^{0.5575}$ and $f(B_j) = \left(\frac{B_j}{1-B_j}\right)^{0.9554}$ are increasing in B_j, for $0 \leq B_j \leq 1$.

Proof: Via proposition **4** we show that $\frac{B_j}{1-B_j}$ is increasing in B_j. For $B_j \in [0, 0.5]$, $\frac{B_j}{1-B_j}$ takes values in $[0, 1]$. Therefore, $\frac{B_j}{1-B_j}$ in some power l $(0 < l < 1)$ is also an increasing function of B_j. ■

Proposition 6: Functions $f(B_j) = \left(\frac{B_j}{1-B_j)}\right)$ is strongly convex for $0 \leq B \leq 0.5$.

Proof: We prove this by investigating the second derivative of this function with respect to B_j.

$$\frac{df(B_j)}{dB_j} = \frac{1}{(1-B_j)^2},$$

and

$$\frac{d^2 f(B_j)}{dB_j^2} = \frac{2}{(1-B_j)^3}.$$

Clearly, $\frac{d^2 f(B_j)}{dB_j^2} \geq 2$ for $0 \leq B \leq 0.5$. Therefore, functions $f(B_j) = \left(\frac{B_j}{1-B_j)}\right)$ is strongly convex. ■

Proposition 7: Functions $f(B_j) = \left(\frac{B_j}{1-B_j)}\right)^{0.9554}$ is strictly convex for $0.04 \leq B \leq 0.5$.

Proof: We prove this by showing that the second derivative of this function with respect to B_j is greater than zero for $0.04 \leq B \leq 0.5$.

$$\frac{df(B_j)}{dB_j} = \frac{0.9554}{(1-B_j)^2(\frac{B_j}{1-B_j})^{0.0446}},$$

and

$$\frac{d^2f(B_j)}{dB_j^2} = \frac{\frac{1.9108(1-B_j)}{\left(\frac{B_j}{1-B_j}\right)^{0.0446}} - \frac{0.0426108}{\left(\frac{B_j}{1-B_j}\right)^{1.0446}}}{(1-B_j)^4}.$$

For $0.04 \leq B \leq 0.5$, function $(1-B_j)^4 \geq 0$. Additionally, for $0.04 \leq B \leq 0.5$, the difference

$$\frac{1.9108(1-B_j)}{\left(\frac{B_j}{1-B_j}\right)^{0.0446}} - \frac{0.0426108}{\left(\frac{B_j}{1-B_j}\right)^{1.0446}} > 0.$$

Therefore, functions $f(B_j) = \left(\frac{B_j}{1-B_j)}\right)^{0.9554}$ is strictly convex for $0.04 \leq B \leq 0.5$. ■

Proposition 8: Functions $f(B_j) = \left(\frac{B_j}{1-B_j)}\right)^{0.5575}$ is strictly concave for $0.04 \leq B < 0.221$ and strictly convex for $0.221 < B \leq 0.5$.

Proof: We prove this by showing that the second derivative of this function with respect to B_j is less than zero for for $0.04 \leq B < 0.221$; and greater than zero for $0.221 < B \leq 0.5$.

$$\frac{d^2f(B_j)}{dB_j^2} = \frac{0.5575\left(\frac{2}{(1-B_j)^2} + \frac{2B_j}{(1-B_j)^3}\right)}{\left(\frac{B_j}{1-B_j}\right)^{0.4425}} - \frac{0.246694\left(\frac{1}{1-B_j} + \frac{B_j}{(1-B_j)^2}\right)^2}{\left(\frac{B_j}{1-B_j}\right)^{1.4425}}$$

The second derivative takes negative values for $0.04 < B_j < 0.221$ and takes positive values for $0.221 < B_j \leq 0.5$.

Therefore, functions $f(B_j) = \left(\frac{B_j}{1-B_j)}\right)^{0.5575}$ is strictly concave for $0.04 \leq B \leq 0.221$ and is strictly convex for $0.221 < B \leq 0.5$. ■

Proposition 9: In an optimal solution to model (**P**), constraints (11) are binding.

Proof: We prove this by contradiction.

Let (X^*, Y^*, B^*) be an optimal solution of (**P**). Let I^* denote the corresponding investment costs, and let S^* denote the set of suppliers selected. The optimal objective function value is $\mathcal{Z}^P(X^*, Y^*, B^*) = \sum_{i \in S^*} \overline{c}_i X_i^* - I^*$.

Let's assume that constraints (11) are not binding at (X^*, Y^*, B^*). Therefore,

$$\sum_{i \in S^*} X_i^* < \frac{(M^{coal} * \rho^b)}{(1/B^* - \alpha)(\rho^b - 0.0044(B^*)^2 - 0.0055)}.$$

Let (X,Y,B) be a feasible solution of (**P**). We create this solution by starting at (X^*,Y^*,B^*) and decreasing the value of B^* by $\epsilon > 0$ so that constraints (11) become binding.

Let's now calculate:

$$\mathcal{Z}^P(X^*,Y^*,B^*) - \mathcal{Z}^P(X,Y,B) = \left(\sum_{i\in S^*} \bar{c}_i X_i^* - I^*\right) - \left(\sum_{i\in S^*} \bar{c}_i X_i^* - I\right) = I - I^* \leq 0.$$

As shown in Propositions **4** and **5**, the investment cost functions do increase with B. Since $B < B^*$, then, $I < I^*$. Therefore,

$$\mathcal{Z}^P(X^*,Y^*,B^*) < \mathcal{Z}^P(X,Y,B).$$

This contradicts the initial assumption that (X^*,Y^*,B^*) be an optimal solution of (**P**). Therefore, at an optimal solution constraints (11) are binding.

■

Proposition 10: Function $f(B_j) = \frac{\rho_j^b B_j}{(1-\alpha_j B_j)(\rho_j^b - 0.0044B_j^2 - 0.0055)}$ is strongly convex for $0 \leq B_j \leq 0.5$, $0 \leq \alpha_j \leq 1$, and $\rho_j^b > 0.0066$.

Proof: We prove this by showing that the second derivative of this function with respect to B_j is greater than zero for $0 \leq B_j \leq 0.5$, $0 \leq \alpha_j \leq 1$, and $\rho_j^b > 0$. The second derivative is:

$$\frac{d^2 f(B_j)}{dB_j^2} = \frac{0.00015488\rho_j^b B_j^3}{(1-\alpha_j B_j)(-0.0055+\rho_j^b - 0.0044B_j^2)^3} + \frac{0.0088\rho_j^b B_j}{(1-\alpha_j B_j)(-0.0055+\rho_j^b - 0.0044B_j^2)^2} =$$

$$= \frac{0.0088\rho_j^b B_j}{(1-\alpha_j B_j)(-0.0055+\rho_j^b - 0.0044B_j^2)^2}\left(1 + \frac{0.0176B_j^2}{(-0.0055+\rho_j^b - 0.0044B_j^2)}\right)$$

For $0 \leq \alpha_j \leq 1$ and $\rho_j^b > 0$, the following holds true: $\left(\frac{0.0088\rho_j^b B_j}{(1-\alpha_j B_j)(-0.0055+\rho_j^b-0.0044B_j^2)^2}\right) > 0$.

Expression $\left(1 + \frac{0.0176B_j^2}{(-0.0055+\rho_j^b-0.0044B_j^2)}\right) > 0$ if $(-0.0055+\rho_j^b - 0.0044B_j^2) > 0$.

The minimum value that expression $(-0.0055+\rho_j^b - 0.0044B_j^2)$ can take is when $\rho_j^b = 0$ and $B_j = 0.5$. In this case, the value of this expression is -0.0066. If $\rho_j^b > 0.0066$, then, $\frac{d^2 f(B_j)}{dB_j^2} > 0$ and function $f(B_j) = \frac{\rho_j^b B_j}{(1-\alpha_j B_j)(\rho_j^b - 0.0044B_j^2 - 0.0055)}$ is strictly convex. ■

Step 1: Initialize λ, UB, n, u, ξ, ϵ, N

Step 2: Let $LB = \mathcal{Z}^Q(X,Y)$

Step 3: Solve subproblems $(\text{SP})_j$ for $j \in C$

Step 4: Compute the upper bound:

$$UB^n = \sum_{j \in J} \mathcal{Z}_j^{SP}(X,Y,B) + \sum_{i \in S} s_i \lambda_i$$

If $UB^n > UB$, then

$UB = UB^n$

$\epsilon = \frac{UB^n - LB}{LB}$

End If

Let $n = n + 1$

Step 5: If $\epsilon \leq 0.01$**, then STOP**

ELSE

Let $u^n = \frac{\xi^n(UB-LB)}{\sum_{i \in S}(s_i - \sum_j X_{ij}^n)^2}$

Let $\lambda_i^n = \lambda_i^{n-1} + u^n(s_i - \sum_j X_{ij})$

Step 6: If $n > N$, then, **STOP**

ELSE go to **Step 3**

Figure 5 Lagrangean relaxation algorithm

Appendix B

Table 9 Set Notations

Sets	
C	the set of coal plants
S	the set of biomass suppliers
L	the set of potential values of β

Table 10 Notations: Decision Variables

Decision Variables	
B_j	represents the percentage of coal (mass basis) displaced in plant $j \in C$ (in %)
X_{ij}	represents the amount of biomass transported from supplier i to plant j (in tons)
Y_j	binary variable that takes the value 1 if $\beta \leq 4$ in plant $j \in C$, and takes the value 0 otherwise
Y_{lj}	binary variable that takes the value 1 if facility $j \in C$ displaced $L_l = \beta\%$ coal, and takes the value 0 otherwise
Z_j	semi-continuous variable that takes the same value as B_j if $Y_j = 1$ and takes the value 0 if $Y_j = 0$

Table 11 Other Notations

Other Notations	
α_j	is equal to $1-(LHV_j^{coal}/LHV_j^{bm})$
β	the percentage of biomass (mass basis) used for cofiring (in %)
β_j	the percentage of biomass co-fired in plant $j \in C$ (in %)
C^{wb}	the conversion factor from MW to BTU/hr (in BTU/(hr*MW)
c_j^{coal}	the unit door price of coal (in %/ton)
c_i^{bm}	the unit purchase cost of biomass from supplier $i \in S$ (in \$/ton)
c_{ij}	the unit transportation cost along arc $(i,j)\ inA$ (in \$/ton)
∇M_j^{coal}	the change in the value of M_j^{coal} (in tons)
EL_j	the efficiency loss of boilers due to co-firing in plant $j \in C$ (in %)
f_j	the utilization rate / capacity factor of plant $j \in C$ (in %)
I_j^{cap}	is equal to $(50,000 * TC_j * f_j * LHV_j^{bm})/LHV_j^{coal}$
I_j^{CAP}	the investment costs in plant $j \in C$ (in \$)
I_j^s	is equal to $I_j^s = 136578 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.9554}$,
I_j^S	the investment necessary for biomass storage in plant $j \in C$ (in \$)
I_j^h	is equal to $I_j^h = 55780 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.9554}$,
I_j^H	the investment necessary for biomass handling in plant $j \in C$ (in \$)
I_j^{cd}	is equal to $I_j^{cd} = 13646 * \left(TC_j * f_j * \frac{LHV_j^{bm}}{LHV_j^{coal}}\right)^{0.5575}$
I_j^{CD}	the investment necessary for compressors and dryers in plant $j \in C$ (in \$)
LHV_j^{coal}	the lower heating value of coal used in plant $j \in C$ (in BTU/ton)
LHV_j^{bm}	the lower heating value of biomass used in plant $j \in C$ (in BTU/ton)
M_j^{coal}	the amount of coal used in plant $j \in C$ (in tons/year)
M_j^{bm}	the amount of biomass used in plant $j \in C$ (in tons/year)
M	a very large number
m_{lj}^{mb}	the amount of biomass necessary to displace $l(\in L)$% of coal in plant $j \in C$ (in tons)
OH_j	the number of operating hours in plant $j \in C$ (in hours/year)
Q_j^0	the initial (before co-firing) annual heating input of plant $j \in C$ (in MW)
Q_j	the annual heating input after co-firing of plant $j \in C$ (in MW)
ρ_j^0	the initial (before co-firing) efficiency rate of plant $j \in C$ (in %)
ρ_j	the efficiency rate after co-firing of plant $j \in C$ (in %)
ρ_j^b	the efficiency rate of boilers in plant $j \in C$ (in %)
ρ_j^{rp}	the efficiency rate of the rest (without boilers) of plant $j \in C$ (in %)
σ_j^p	is equal to $(c_j^{coal} * \frac{LHV_j^{bm}}{LHV_j^{coal}})$
S_j^p	the total savings due to reducing the amount of coal purchased in plant $j \in C$, (in \$/year)
σ_j^t	is equal to $(11 * LHV_j^{bm})/(C^{wb})$
S_j^{tax}	the total savings due to production tax savings in plant $j \in C$, (in \$/(ton*year))
TC_j	coal plant $j \in C$ nameplate capacity (in MW)