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TWO ESSAYS ON FINANCIAL ECONOMETRICS

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TWO ESSAYS ON FINANCIAL ECONOMETRICS

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ABSTRACT

The first paper examines the properties of the realized volatilities of US Dollar / Canadian Dollar spot exchange rate covering a time span of about three years and then the deseasonalized volatilities are estimated and forecasted using a fractionally-integrated model. The key feature of the realized volatilities is that they are model-free and also approximately measurement-error-free. Usually a U-shaped pattern of the intraday volatilities should be observed due to opening-closure effects in the global market. I do not see a typical U-shaped pattern in the intraday volatilities for US Dollar / Canadian Dollar. The reasons are given in this paper. I use ARFIMAX model to estimate and forecast the deseasonalized volatilities and the results are promising.

The second paper proposes a time series based trading strategy for "pairs trading". Pairs trading is one of the oldest statistical arbitrage strategies and has been proved to be successful on Wall Street. Most academic studies on pairs trading focus on pair selection or optimal threshold comparison. This is the first paper to introduce time series methodology into research of pairs trading. The dynamics of the spread between two stocks in a pair are tested and examined. A time series "dynamic threshold method" is proposed in this paper and the trading strategy based on this method improves the excess return of traditional naïve pairs trading model significantly.

DEDICATION

To my parents, Qinghai Geng and Guiqing Liu and my wife, Ye Kong

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I would like to thank members of my dissertation committee: Dr. Olga Isengildina-Massa (chair), Dr. William Bridges, Dr. Chris Kirby, and Dr. James Nyankori for their helpful comments and encouragement.

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1. Introduction

Profit earning is the purpose of investors and they need to estimate the risk of the investment and make decisions with the respect to this estimation. Risk in financial market is closely connected to volatility and therefore volatility in financial markets has been one of the most studied topics. The role of volatility can be found in financial asset pricing, financial hedging, risk management and other related fields. However unlike price, volatility is unobservable and only its realization can be measured ex post. For example, volatility is the only variable that cannot be observed in the famous Black-Sholes model. Therefore, reliably measuring and forecasting volatility is very important for both academic research and practical use.

Volatility is defined as the standard deviation of the continuously compounded returns of a financial instrument with a specific time horizon and based on this definition the model-free unbiased measure of volatility is the square root of the sample variance of returns. For example, weekly volatility may be estimated using daily returns over a week and thus one can construct a time series of model-free variance estimates. When intraday returns are available, daily volatility can be estimated the same way.

As an alternative, the model-free unbiased estimates of the *ex post* daily volatility can be proxied by daily squared returns. Lopez (2001) used daily squared return which was calculated from daily closing price, to proxy daily volatility. This method was criticized by Andersen and Bollerslev (1998) and Christodoulakis and Satchel (1998)..

Both researches found that using squared returns as proxy of volatility will lead to low R^2 and undermine the inference.

The indirect way to measure volatility is to use implied volatility which can be generated from the market price of the option based on an option pricing model (i.e. the Black-Scholes model). In other words volatility is implied in the option price, given a particular option pricing model. The Black-Scholes option pricing model states that option price is a function of the pricing of the underlying asset, the risk free interest rate, the strike price, and volatility of the underlying asset in the defined period. This volatility can be calculated in the way of "reverse-engineering" from the price of the option given the option price is observable. Implied volatility is a forward-looking measure and it measures the volatility of the underlying asset from now until the option expires. It differs from historical volatility because the latter is calculated from known past prices of a security. The problem with implied volatility measured with Black-Scholes model is that most option pricing models assume that logarithmic stock returns follow normal distribution. At the same time more and more research shows that financial asset returns have fat tails (Engle (1982), Engle (2001), Poon and Granger (2003)).This weak assumption in the option pricing models makes the accuracy of implied volatility questionable (i.e. volatility smiles).

Volatility measured based on square returns is called historical volatility and it uses the historical information to capture the main effect. Implied volatility is called option based forecast and it is calculated from option prices instead of historical information. These two methods are different in both assumptions and use of information.

Volatility analysis based on high frequency data is not a new topic. Merton (1980) addressed that the variance over a fixed horizon can be estimated as the sum of squared realizations if the data are available at a sufficiently high sampling frequency. More recently, Anderson, Bollerslev, Diebold, and Labys (2001), found that higher data frequency can take care of the problems found in traditional historical volatility calculation and they introduced a new name for historical volatility calculated using high frequency data: realized volatility. The basic idea of realized volatility is that a reliable measure of the sample variance can be proxied by the summation of squared returns over the relevant horizon. When the data frequency approaches infinity, it is demonstrated that as the theory of quadratic variation proves, the realized volatilities are not only model-free, but also measurement-error-free (ABDL (2001)). For this extreme case instead of saying that realized volatility is proxied, we say realized volatility is "observed". In practice, although we cannot obtain infinite high frequency data, realized volatility still approaches the underlying integrated volatility when the data frequency is high enough. The approach used in this paper is to calculate realized volatility from the sample of high frequency returns.

Motivated by the work of Anderson, Bollerslev, Diebold, and Labys (2001), hereafter ABDL, and ABDE (2000 and 2001), I examine the volatilities of US Dollar / Canadian Dollar, hereafter USD/CAD, spot exchange rate over a three-year period. I checked the properties of the realized volatilities for USD/CAD. Basic observations in this paper are consistent with previous studies. For example, the realized volatilities of USD/CAD are skewed and leptokurtic, but the logarithms of realized volatilities are approximately Gaussian. I also find long-memory effect in the realized volatilities, and a fractionally-integrated long-memory model is used to estimate and forecast the realized volatilities.

This paper differs from the literature discussed above in the following aspects. Most studies use arbitrarily chosen fifteen-minute or thirty-minute interval to generate realized volatilities. Because different assets in different financial markets may have different properties, an arbitrary interval cannot guarantee the best estimation. And the first objective of this study was to use a method called summation of cross multiplications (SCM) to select the optimal interval.

The second objective was to consider the pattern of returns of USD/CAD and compare it to the U-shaped patterns (intraday periodicity) typically addressed in the opening-closure theories (see e.g. Foster and Viswanathan, 1990; Son, 1991; Brock and Kleidon, 1992). The plot of the average returns for USD/CAD does not show a typical U-shaped pattern and no previous study has ever considered this problem to the best of my knowledge. In this paper I give an explanation for this unique phenomenon. Understanding the intraday periodicity can help us deseasonalize the realized volatilities and thus provide better forecasting.

The third objective was to develop a forecasting mechanism that best fits properties of the data. In the study I find that there is long-memory process[1] in the logarithmic realized volatilities according to fractional integration and cointegration test. Leverage effect[2] is also detected based on regression analysis and scatter plots between return and realized volatilities. To capture all these properties I use a modified

[1] A long-memory process is one in which the autocorrelation at a lag k decays at a rate slower than the usual rate of k-1.
[2] Leverage effect refers to a negative correlation between past returns and future volatility.

Autoregressive Fractionally Integrated Moving Average Model with Explanatory variables (ARFIMAX) model to estimate the realized volatilities.

The rest of the paper is organized as follows: In section 2 I do a brief literature review on volatility measurement and estimation; in section 3 I discuss the computation of realized volatilities based on the optimal interval using high frequency USD/CAD data. Then the properties of returns and standardized returns, realized volatilities and logarithmic realized volatilities are studied; in section 4 I study the intraday seasonality of USD/CAD spot exchange rate. A deseasonalized series are generated for future estimation; and in section 5 I apply ARFIMAX model to estimate and forecast the deseasonalized volatilities I obtained in the previous section. Section 6 is the conclusions.

2. Conceptual Framework

In finance, volatility refers to the standard deviation or variance of the return series. The discrete form of volatility is calculated as following (Poon and Granger, 2003):

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_t - \bar{r})^2 \qquad (1)$$

where N is the number of returns during the time period , $\bar{r}$ is the sample mean of N returns and r_t is defined as $\frac{p_t - p_{t-1}}{p_{t-1}}$ and it is the specific return at time[3] t. Stephen (1997) noted that since sample mean is not an accurate estimate of true mean when

[3] Before the availability of high frequency intraday data, volatility is calculated based on daily returns. For example N equals 5 if we calculate weekly stock volatility and r_t is the return at day t.

sample size is small, variance calculated around zero instead of sample mean could increase volatility estimation accuracy.

The continuous time analogy of discrete volatility is called integrated volatility. It measures the speed of the price change compared to a standard wiener process (Hull (2003)). The change of the price is decomposed as a standard wiener process with variance of σ plus the drift across time:

$$dp_t = \sigma dW_{p,t} + \mu dt \tag{2}$$

or

$$\sigma^2 = (\frac{dp_t - \mu d_t}{dW_{p,t}})^2$$

where dp_t is the continuous form of price changes, $dW_{p,t}$ is a standard wiener process, μ is the drift and dt is the change of time. In (2), price is the only variable that can be observed at time *t*, and volatility is a latent variable that scales the stochastic process $dW_{p,t}$ continuously through time.

With the availability of high frequency intraday data, let $p_{n,t}$ denote the price of an asset at time $n \geq 0$ at day t. n = 1,2,…,*N* , it is the number of observed prices in a day and *N* equals to 1440 if prices are recorded every minute. t=1,2,…,*T* and it is the number of active trading days in sample. Note that when *n*=1, p_t is simply daily price of the asset (normally recorded as the closing price). The continuously compounded returns with N observations per day is given by[4],

[4] The mathematical definition of a return is: $\frac{p_t - p_{t-1}}{p_{t-1}}$, most researchers use $(\ln(p_t) - \ln(p_{t-1}))$ for continuously compounded returns.

$$r_{n,t} = \ln(p_{n,t}) - \ln(p_{n-1,t}) \tag{3}$$

where $r_{n,t}$ is the n th continuously compounded return at day t.

To make the notation simple, when n=1 I simply ignore the subscript n and $r_t = \ln(p_t) - \ln(p_{t-1})$ where t= 2,…,T. In this case, r_t is the time series of daily returns, the following assumptions confirms to

In (3):

(a) $E(r_{n,t}) = 0$

(b) $E(r_{n,t} r_{m,s}) = 0$ for $n \neq m$ and $t \neq s$

(c) $E(r_{n,t}{}^2 r_{m,s}{}^2) < \infty$ for n,m,s,t

$r_{m,s}$ is the m th continuously compounded return at day s where $n \neq m$ and $t \neq s$.

Assumption (3a) implies that the mean return is zero and this follows from the fact that the log prices, $\ln p_t$, follow an *i.i.d.* random walk process without a drift shown as below,

$$\ln(p_{n,t}) = \ln(p_{n-1,t}) + \varepsilon_{n,t} \qquad where\ \varepsilon_{n,t} \mid I_{t-1} \sim i.i.d.(0, \sigma_t^2) \tag{4}$$

Following (4), $r_{n,t} = \ln(p_{n,t}) - \ln(p_{n-1,t}) = \varepsilon_{n,t}$ and thus, $E\ (r_{n,t})$ =$E\ (\varepsilon_{n,t})$ =0. Assumption (3b) follows from the fact that $\varepsilon_{n,t}$ are *i.i.d.* and from (a) which gives us $E(r_{n,t} r_{m,s})$=$E(\varepsilon_{n,t}\varepsilon_{m,s})$= 0. Assumption (3c) states that the variances and co-variances of the squared returns exist and are finite. This follows from the fact that $E(r_{n,t}{}^2 r_{m,s}{}^2)$=E($\varepsilon_{n,t}{}^2\varepsilon_{m,s}{}^2$)<$\infty$ for n,m,s,t.

From (4), the continuously compounded daily return (Campbell, Lo, and Mackinlay, 1997) is given by,

$$r_t = \sum_{n=1}^{N} r_{n,t} \tag{5}$$

and therefore the daily squared return is calculated as

$$r_t^2 = (\sum_{n=1}^{N} r_{n,t})^2 = \sum_{n=1}^{N} r_{n,t}^2 + \sum_{n=1}^{N}\sum_{m=1}^{N} r_{n,t} r_{m,t} = \sum_{n=1}^{N} r_{n,t}^2 + 2\sum_{n=1}^{N}\sum_{m=n+1}^{N} r_{n,t} r_{m-n,t} \tag{6}$$

In (6) the squared daily return can be decomposed into two components: the daily sample variance and twice the sum of $N - 1$ daily sample autocovariances (measurement error). Note that $\sigma_t^2 = Var(r_t) = E(r_t^2)$ since we have $E(r_t) = 0$ and assumption (3b). Under these conditions the sample variance of high-frequency returns is a valid estimator of the daily population variance σ^2 and this estimator is unbiased. According to Barndor-Nielsen and Shephard (1999) and Karatzas and Shreve (1988), it follows $P\lim_{N\to\infty} \sum_{n=1}^{N} r_n^2 = \sigma^2$ by the theory of quadratic variation. Thus, the sum of the intra-daily squared returns is an unbiased and consistent estimator of the daily population variance. The measurement error in (6) can be made arbitrarily small by summing sufficiently many high-frequency squared returns if microstructure friction effects (such as bid-ask spreads, liquidity ratios, turnover, and asymmetric information) [5] are neglected. The sum of the intra-day squared returns is known as the realized volatility h_t^2 (also called the realized variance by Barndorff-Nielsen and Shephard (2002)).

[5] According to Zhou (1996), there are different sources of microstructure noises: For example, there is a fighting-screen effect. To keep their name on the Reuters screen, traders keep updating their quotes. The new update is often slightly different from the previous quotes even if the market level has not changed. Micro-activities are another contribution to the noise. Small typographical errors or delayed quotes are all sources of noises.

We can also get the estimated error,

$$E(\sum_{n=1}^{N} r_n^2 - \sigma^2)^2 = \frac{\sigma^4}{N}(K_n - 1)(1 + 2\sum_{n=1}^{N-1}\frac{N-n}{N}\rho_n) \tag{7}$$

In (7), K_n is the kurtosis of r_n and ρ_n is the nth autocorrelation coefficient of r_n^2 (Karatzas and Shreve, 1988). From (7) we can see that error will decrease when the frequency of the dataset increases (*N* increases). In theory if we want to get the best estimation of the volatilities, we need to use the highest dataset frequency and the smallest time interval. That means that given enough observations for a given trading day, the realized volatility can be computed and is a model-free estimate of the conditional variance which is usually generated in models like ARCH model. In the real world, extremely high frequency data may not be a good choice for research: firstly prices do not follow normal distribution when the data frequency is too high (i.e. the time interval is less than five minute), secondly Anderson, Bollerslev and Das (1998) found that because of microstructure friction effects in dataset, the volatility estimates based on the high frequency model-free method can be very noisy in practice The properties of the realized volatilities are discussed in ABDL (2001). In particular, the authors found that the realized volatility is a consistent estimator of the daily population variance σ_t^2 .

3. Empirical Analysis

3.1 Data

My empirical analysis focuses on the spot exchange rates for the U.S. dollar and the Canadian dollar. The raw data consist of all one-minute interval prices for USD/CAD displayed on the ForeXite during the sample period, January 2, 2004 through April 24,

2007. There are 864 effective trading days (weekends are not included) and 1,223,644 observations. In this paper, all returns are computed as the first difference in the regularly time-spaced (1 minute) log prices of the exchange rate index: $r_t = \ln p_t - \ln p_{t-1} = \ln(p_t / p_{t-1})$. Because the exchange is open 24 hours, the first intraday return is the first difference between the log price at 00:01 am and the log price at 23:59 pm the day before.

Figure 1 is generated using one-minute prices for January 02, 2007. From the figure we can see that prices fluctuated dramatically. This is due to microstructure effects (Andersen, Bollerslev and Das (1998)) in the dataset. Prices can be separated as a fundamental component and a microstructure noise component. The volatility of microstructure noise component increases as the data frequency increases. According to (6), not only the frequency N but also the autocorrelation coefficient ρ_n of the return series affects the estimated error. The fluctuation of prices means that the autocorrelation coefficient for the return series is negative and large in magnitude. In practice the estimated error maybe too large due to the significantly increased ρ_n in a high frequency dataset. Therefore selection of the best estimation frequency should be based on the trade off between standard estimated error and error from microstructure friction effects in the dataset.

Previous research (ABDL (2003)) suggests that the use of equally-spaced thirty-minute returns strikes a satisfactory balance between the accuracy of the continuous-record asymptotic underlying the construction of the realized volatility measures on one hand, and the confounding influences from market microstructure frictions on the other. Is this always the case? Does the thirty-minute interval fit all the situations? Some scholars criticized this arbitrary selection and proposed different approaches to select the best interval. For example Andersen and Benzoni (2008) suggested using volatility signature plot to find the best interval that can assess the trade off. Huang and Tauchen (2005) suggested dealing with the problem using alternative QV estimators which is less sensitive to microstructure friction effects . In this paper I create a new variable which is the sum of cross multiplications every N (here N denotes the number of intraday periods) observations. The formula is as follows,

$$SCM = \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} r(n) * r(m) \qquad (8)$$

In (8) we can see that there are $N*(N\text{-}1)$ cross multiplications for each day. Then the summation of this $N*(N\text{-}1)$ cross multiplications (SCM) will be calculated for each day using the datasets with different intervals (different N) and the dataset that gives the smallest mean of SCM will be the dataset with the "best" interval.

I plot the means of SCM in Figure 2 with time intervals on the horizontal axis. From Figure 2 we can see that when I use one-minute returns, the mean of summation of cross multiplications is the highest. Then after a steep drop from -4.53901E-12 to -2.63805E-06, the means start to be consistent from the point where the interval is ten-minute. But when the interval is longer than eighteen-minute the means begin to increase slightly. This shows that thirty-minute interval used in previous researches (i.e. ABDL (2001)) is not the best frequency for this particular dataset. From this figure we can see that a time interval between the ten-minute and eighteen-minute is suitable for my estimation. In the remainder of the paper, I chose the return series with a fifteen-minute interval where the mean of the cross multiplication for the series is about -2.6002E-06.

3.2 Properties of exchange rate returns and realized volatilities

Returns

Table 1 shows the descriptive statistics of the returns for the exchange rate. The mean return for the exchange rate during the time span I studied is negative, -.0001672, which is very close to zero. The standard deviation of the returns is .0048404. From the Table 1 we can see that it has a positive estimate of skewness of .0585405 which indicates that the distribution of the returns is not symmetric and is actually slightly right-skewed. The estimate of the sample kurtosis is above the normal value of 3 meaning that the distribution of the returns is leptokurtic. All above findings are consistent with those found in ABDE (2001).

To test the joint significance of the first 20 auto-correlations of the returns, a standard Ljung-Box portmanteau test[6] was performed and the results are shown in the right panel of Table 1. The reported p- value of the corresponding $Q(20)$ statistics is .0910 which indicates we barely fail to –at the 10% significance level- reject the null hypothesis of zero autocorrelation, suggesting a low persistence for the return series[7]. While the p- value of the $Q^2(20)$ for the squared returns indicates a rejection of the null hypothesis that there is no serial correlation, which means that there is some volatility clustering effect in the returns. All the above results are consistent with the extensive literature documenting heavy tails and volatility clustering in asset returns, dating at least to Mandelbrot (1963) and Fama (1965).

[6] Other tests for presence of autocorrelation are Durbin–Watson statistic (first order autocorrelation test) and Breusch–Godfrey test.

[7] According to ABDL (2002), under the null hypothesis of white noise, the reported Ljung-Box statistics are distributed as chi squared with twenty degrees of freedom. The five percent critical value is 31.4, and the one percent critical value is 37.6

Standardized returns are obtained by dividing the original returns with their corresponding realized standard deviation h[8]. In this paper they are expressed as:

$$std(r) = r / h \tag{9}$$

where r is the original return and h is the realized standard deviation.

The results of the descriptive statistics are shown in the lower panel of Table 1. The mean for the standardized returns are also negative -.0002832 which is close to the mean of the raw returns I found. And the standard deviation for the standardized returns is larger than that of the raw returns. Although the coefficient for the skewness is still positive, the value has been decreased and is closer to zero. The coefficient for the kurtosis of the standardized returns now has a value of 2.72022 which is closer to the normal value of 3 compared to 3.585104 of the raw returns. We can also see this from Figure 3 which shows kernel densities of the raw returns and standardized returns respectively. Both the table and the figure show that standardized returns are closer to normal compared to raw returns. This finding is consistent with ABDE (2001) and ABDL (2001, 2003), who show both the stock returns and exchange rate returns standardized by their respective realized standard deviations are closer to normal and can be treated as approximately Gaussian.

The results of the Ljung-Box test are also shown in the lower panel of Table 1. The value is 23.9372 and is not significant. As a result, I conclude that there is no or a very weak persistence in the standardized returns. As regards the test for autocorrelation in squared standardized returns, I find that the $Q^2(20)$ statistics is 23.2346 and is not significant suggesting that we fail to reject the null hypothesis that there is no serial

[8] Standardization is calculated by subtracting the center (usually the population mean) from the data and then dividing the difference by population standard deviation. In this research I standardize the returns to mean zero.

correlation. Because the $Q^2(20)$ for standardized returns is only about half of that for raw returns, we know that the volatility clustering effect was reduced.

Realized Volatilities

Standardized returns were used to calculate realized volatility using equation (9). As we can see in Table 2, the mean of realized volatilities is .5268207. The sample skewness coefficient is positive meaning that the distribution of the realized volatilities is skewed to the right. This can be confirmed from Figure 4(a). From Table 2 we can see that the sample kurtosis coefficient is 5.38737 which is larger than the normal value of 3 implying that the distribution is highly leptokurtic.

The results for the logarithmic transformation of the realized volatilities are shown in the lower panel of Table 2. As we can see in the table, the skewness is -.4933379 which reduced remarkably in magnitude compared to that of the original realized volatilities. Because -.4933379 is negative, we can see a relatively symmetric distribution with left skewness for the logarithmic realized volatilities in Figure 4(b). The kurtosis for the transformed realized volatilities is large and that means the distribution is also highly leptokurtic. This is confirmed from the Figure 4(b). To be consistent with previous studies, logarithmic realized volatilities are used in this paper since the logarithmic series is closer to normal and normality is going to be critical for later estimation[9].

Early study of the long-memory, or fractionally-integrated, effects in volatilities by Robinson (1991) and subsequent studies suggest the empirical relevance of long

[9] The main disadvantage of taking log is to lose some useful information of the original dataset. I use logarithmic transformation in this paper because in ARFIMAX model, normality is an important assumption for the linear regression part.

memory in asset return volatilities. Other studies (see, for example, Renault, 1997; Comte and Renault, 1998; and Bollerslev and Mikkelsen, 1999) conclude that long-memory processes also help to explain anomalous features in options such as volatility smile even for long dated options.

In the last column of Table 2 I report estimates of the degree of fractional integration, obtained using the Geweke and Porter-Hudak (1983) (GPH) log-periodogram regression estimator as formally developed by Robinson (1995). If the volatility is a long-memory process it is neither stationary (I[0]) nor is it a unit root (I[1]) process; it is an (I[d]) process, with d[10] (fractional integration parameter) a real number. The estimate of d is significantly greater than zero meaning there is a significant long-memory effect in the logarithmic volatilities and therefore we need appropriate model to catch this effect in future estimation and forecasting.

Returns and Realized Volatilities

It is interesting to check the relationship between returns and realized volatilities. Pagan and Schwert (1990) and Engle and Ng (1993), among others, have documented asymmetries in the relation between news and volatilities. Both papers concluded that good and bad news have different impact for future volatility. Most papers thereafter found that a lagged negative return tends to increase subsequent volatility by more than would a positive return of the same magnitude. This phenomenon is known as the 'leverage' or 'news' effect.

[10] The fractional integration parameter (d) is calculated based on the spectral regression method introduced by Geweke and Porter-Hudak (1983).

In Figure 5, I have two scatter plots addressing the relationship between lagged returns and realized volatilities. According to the *p*-values reported in the plots, both regressions have slope significantly different from zero. The non-zero coefficients mean that volatility increases for each unit increase in lagged return. More specifically, both plots suggest significant leverage effects: negative lagged returns yield different volatilities than lagged positive returns. The reason why lagged returns have effects on current volatilities is because it usually takes time for market participants to react to previous news.

4. Intraday Periodicity

4.1 Intraday return periodicity

The most important reason why I use high frequency data is that high frequency data contain more information than daily data and therefore I can have detailed information to study intraday phenomena which is critical in modeling and forecasting volatilities. Intraday seasonality, a highly persistent conditionally heteroskedastic volatility component, is one of the most important intraday phenomena which can be traced using high frequency data. A typical U-shaped pattern of intraday volatilities has been observed in several previous studies, including Baillie and Bollerslev (1991), Harvey and Huang (1991), Dacorogna et al. (1993), Cornett et al.(1995), Bollerslev and Ghysels (1996) and others.

Like in all previous studies, I found very strong intraday seasonality in my dataset. In Figure 6[11], the first graph shows the average returns over every fifteen-minute

[11] The dataset we used in this paper is generated based on Greenwich Mean Time(GMT). In figure 5 we rescaled the data into EST time which is easy to analyze.

interval and it does not have a particular pattern during a trading day. The second graph in Figure 6 plots the absolute value of the average returns (absolute returns) over every fifteen-minute interval and it shows some particular pattern which may be important for further study. This is consistent with the observations of previous studies such as Andersen and Bollerslev (1997a) and (1997b) where absolute returns showed more information than raw returns. The third graph in Figure 6 plots the five-minute moving average of absolute returns over every fifteen-minute interval. This graph shows a clear heavy tailed M-shaped intraday seasonality of the volatilities. This observation is obviously not consistent with any of the previous studies where either a U-shaped or a double U-shaped pattern was observed.

There are two possible reasons: first, most of the previous studies were about stock markets and the U-shaped or double U-shaped returns are due to the significant strategic interaction of traders around market openings and closures (see e.g. Foster and Viswanathan, 1990; Son, 1991; Brock and Kleidon, 1992). While foreign exchange market is a 24-hour market and there is no such significant opening or closure effects in the daily returns. Secondly, USD/CAD exchange rate market is a unique foreign exchange market due to the geographical locations of the traders. Not like other most widely traded currencies for which traders are located in Europe, Asia or North America, the traders for USD/CAD are mostly located in North America.

Let us take a closer look at the graph of the returns. The volatilities start out at a relatively low level and climbs up at a relatively low speed until interval 32 (EST 8:00AM). From interval 32, the volatility starts to take off corresponding to the opening of the North American market. The strong drop between interval 40 and 50 corresponds to the closure of the European markets. The activity then picks up during the afternoon session of the North American market until interval 68 (EST 5:00PM). Then after the North American market is closed, the volatilities flats at a relatively low level in the rest of the day. Therefore my result is consistent with most of the previous studies although its "unique" looking.[12]

4.2 Seasonal Adjustment

Previous studies, including Hsieh (1989), Baillie and Bollerslev (1991), Poon and Granger (2003), among others, have suggested that ARCH/GARCH-related models can adequately characterize volatility persistence in daily exchange rate changes. However, it is also noted that classical ARCH/GARCH models without seasonality adjustment may not be able to successfully capture temporal persistence in the case of high-frequency returns, as argued in Andersen and Bollerslev (1997a) and Andersen and Bollerslev (1998) and Martens et al. (2002). Because standard parametric models of volatility are unable to capture temporal persistence and intraday seasonality jointly when applied to high-frequency return data, I need to perform seasonal adjustment before my estimation.

Taylor and Xu (1997) proposed to use the appropriate average of the squared returns over all trading days. Let $r_{n,t}$ denote the *n*th intraday return on day *t*, and suppose we have *T* days and *N* intraday periods. Then we have the seasonal variance c_n^2 as follows

[12] Daylight savings time is observed in Europe and North America, but not in East Asia. Andersen and Bollerslev (1994) analyzed this effect and concluded that it gives rise to a one hour difference in the peaks associated with the regular release of U.S. macroeconomic announcements at 08.30 a.m. corresponding to interval 162 for winter time and interval 150 for summer time. Another effect, day-of-the-week effect, was also studied by Ederington and Lee (1993) and Harvey and Huang (1991). Their conclusion is that macroeconomic announcement effects could have an impact on the average volatility in Friday morning trading in the U.S market.

$$c_n^2 = \frac{1}{T}\sum_{t=1}^{T} r_{n,t}^2 \, (n = 1 \ldots N) \tag{10}$$

Andersen and Bollerslev (1997a, 1998) used the logarithm of the squared returns to help estimate seasonal patterns. The assumption is that volatility is the combination of the seasonal volatility and the nonseasonal component. Based on their definition the seasonal variance estimate is given by

$$c_{n,t}^2 = \exp[\frac{1}{N_t}\sum_{s \in S_t} \ln(r_{n,s} - \bar{r})^2] \tag{11}$$

where $\bar{r}$ is the average return based on the whole sample.

When we get the seasonal terms we can filter the returns using these terms based on the formula below,

$$\tilde{r}_t \equiv \bar{r}_{n,t} \equiv \frac{r_{n,t}}{c_{n,t}} \tag{12}$$

After my estimation using the deseasonalized returns, we can then transform the deseasonalized volatility forecasts back to the forecast for the original returns by multiplying the volatility forecast by the appropriate seasonal term, $c_{n,t}$. And this gives us a true return (versus the deseasonalized return in estimation step) with periodicity taken care of in the estimation step[13].

5. Prediction

[13] As mentioned previously, pervasive intraday periodicity in the return series has strong impact on the dynamic properties of high frequency volatility. The patterns in this dataset (Figure 6) are so distinctive there is a strong case for taking the periodicity into account before attempting to model the dynamics of volatility.

Below is the summary of my main findings of previous sections: (1) the distribution of realized volatilities is asymmetric and leptokurtic, while the distribution of the logarithmic realized volatilities is approximately Gaussian; (2) according to the GPH test there is long-memory process in the logarithmic realized volatilities; (3) leverage effect is detected based on regression analysis and scatter plots between return and realized volatilities; and (4) there is a strong intraday seasonality in the return series. Based on the above properties of the volatilities, I am going to use alternative ARFIMAX models to estimate and forecast the volatilities in this chapter. The estimation performance will also be evaluated in this chapter.

5.1 ARFIMAX model

Model

Ebens (1999) proposed the ARFIMAX model and estimated the realized volatilities of Dow Jones Industrial Average (DJIA) portfolio using this model. His original model is as follow,

$$(1-L)^d(1-\phi(L_P))\ln(h_t^2) = \omega_0 + \omega_1 r_{t-1}I^- + \omega_2 r_{t-1}I^+ + (1+\theta(L_P))\varepsilon_t \qquad (13)$$

where $\varepsilon_t \sim i.i.d\ \ N(0,\sigma^2)$, L is the back shift operator[14], $\phi(L_q) = \sum_{i=1}^{p}\phi_i L^i$ and $\theta(L_q) = \sum_{i=1}^{q}\theta_i L^i$. Realized volatilities are denoted by h_t^2, the indicator $I^-(I^+)$ takes value of one when return $r_{t-1} < 0 (r_{t-1} \geq 0)$ and is zero otherwise. This model was generated based on the classical ARMA (p, q) model where the ARMA coefficients are ω_0,

[14] $(1-L)^d(\ln(h_t^2)$ is a fractionally differenced process. L is the back shift operator such that $L(\ln(h_t^2)) = \ln(h_{t-1}^2)$

$\phi(L_P)$ and $\theta(L_P)$. The new items in the model are a fractional integration parameter (*d*) to capture the slow hyperbolic decay in the sample autocorrelation function; lagged negative (ω_1) and positive (ω_2) returns to capture the leverage effect in the distribution of $\ln(h_t^2)$.

In this paper, I use a modified ARFIMAX model which is given below,

$$(1-L)^d(1-\phi(L_p))(\ln h_t^2 - K - k_1 r_{t-1} I^- - k_2 r_{t-1} I^+) = (1+\theta(L_p))\varepsilon_t \tag{14}$$

Compare my revised model with the original ARFIMAX model we can see the difference between two models: in model (14) I do regression first and then estimate the fractional integrated moving average while in the original model they estimate the moving average first and then do the regression. The general form of my modified model can be written as,

$$(1-L)^d(1-\phi(L_p))(y - X\beta) = (1+\theta(L_p))\varepsilon_t \tag{15}$$

Ebens (1999) used conditional sum-of-squares maximum likelihood (SSML) estimator (advocated by Hosking 1984) to estimate the coefficients of the model. In this paper I use modified profile likelihood method (MPL) to estimate the model. An and Bloomfield (1993), and Doornik and Ooms (1999) proved, based on Monte Carlo simulation, that MPL will eliminate the negative bias commonly found in SSML.

We have the likelihood function,

$$\log L(d,\phi,\theta,\beta,\sigma_\varepsilon^2) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log|\Sigma| - \frac{1}{2}\log z'\Sigma^{-1}z \tag{16}$$

where $z = y - X\beta$, Σ is the auto covariance matrix of $y = (y_1,\ldots,y_T)'$. Because we know that auto correlation matrix $R = \sum\frac{1}{\sigma_z^2}$, we can rewrite (16) into,

$$\log L(d,\phi,\theta,\beta) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log|R| - \frac{T}{2}\log\sigma_\varepsilon^2 - \frac{1}{2\sigma_\varepsilon^2}(z'R^{-1}z) \qquad (17)$$

If we take the derivative of (17) with respect to σ_ε^2, and let it equal to zero, then we have,

$$\log L(d,\phi,\theta,\beta) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log|R| - \frac{T}{2} - \frac{T}{2}\log(T^{-1}z'R^{-1}z) \qquad (18)$$

We can also take the derivative with respect to β and get,

$$\log L(d,\phi,\theta) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log|R| - \frac{T}{2} - \frac{T}{2}\log(T^{-1}\hat{z}'R^{-1}\hat{z}) \qquad (19)$$

And then we have the modified profile likelihood for ARFIMAX (*p, d, q*) as follows,

$$\log L(d,\phi,\theta) = -\frac{T}{2}(1+\log(2\pi)) - (\frac{1}{2}-\frac{1}{T})\log|R| - \frac{T-k-2}{2}\log(T^{-1}\hat{z}'R^{-1}\hat{z}) - \frac{1}{2}\log|X'RX|$$

where k is the degree of freedom. (20)

Cheung and Diebold (1994) found that most of the errors in fractional-integrated estimation are from the mean. If the sample is not very large, we can use the average of the sample to replace the mean in the likelihood function and get a better estimation. Following this approach, we use the below modified model for estimation,

$$(1-L)^d(1-\phi(L_p))(\ln h_t^2 - \hat{\mu} - k_1 r_{t-1}I^- - k_2 r_{t-1}I^+) = (1+\theta(L_p))\varepsilon_t \qquad (21)$$

where $\hat{\mu}$ is the average of $\ln h_t^2$.

Ebens (1999) only estimated the ARFIMAX model without autoregression term, or FIMAX model. I estimate model (21) using the likelihood function (20). ARFIMAX (1,d,1,X) is the full model where "1" is the first order autoregression term, "d" is the fractional integration parameter, “1” is the first order moving average term, and “X” means there are exogenous variables in the model.

And in this paper I estimate all the six alternative models and compare their performance. In the end I select the best model for my forecasting. The six alternative models are: ARFIMAX (0, d, 0), ARFIMAX (1, d, 1), ARFIMAX (0, d, 0, X), ARFIMAX (0, d, 1, X), ARFIMAX (1, d, 0, X), and ARFIMAX (1, d, 1, X). The first model is a FI model, the second model is the well-known ARFIMA model and the other four models are the exhaustive possibilities with explanatory variables fixed in the model.

In Table 3 we can see that all the fractional-integrated coefficients are significant with the minimum of .267062 and the maximum of.480377. The significant coefficients mean that there is strong long-memory effect in the volatilities. The leverage coefficients k_1 and k_2 are both significant suggesting that there is strong leverage effect in the series. Above observations confirmed that ARFIMAX is a suitable model for my estimation.

Comparing the results in Table 3 we can see that model (0, d, 0, X) has the lowest AIC value and outperforms all other five models. Model (0, d, 0, X) refers to an ARFIMAX model without autoregression term and moving average term.

Forecasting

In this section, I use the selected best model to forecast the volatilities in the next period. The forecasting method based on ARFIMAX (0, d, 0, X) is shown as below,

$$(1-L)^d (\ln(h_t^2 - \hat{\mu} - k_1 r_{t-1} I^- - k_2 r_{t-1} I^+) = \varepsilon_t \qquad (22)$$

let $\pi_t = \ln h_t^2 - \hat{\mu} - k_1 r_{t-1} I^- - k_2 r_{t-1} I^+$, then $\pi_t + b_1 \pi_{t-1} + ... + b_{t-2}\pi_2 + \tilde{\prod} = \varepsilon_t$, where b_t are coefficients from the model and $\tilde{\Pi}$ is the summation of the residuals. Because $E(\pi_t) = 0$ then $\tilde{\Pi} = 0$. And then we have,

$$\begin{aligned}\pi_t &= \ln h_t^2 - \hat{\mu} - k_1 r_{t-1} I^- - k_2 r_{t-1} I^+ \\ &= -b_1 \pi_{t-1} - b_2 \pi_{t-2} - ... - b_{t-2}\pi_2 + \varepsilon_t\end{aligned}$$

and

$$\ln h_t^2 = \hat{\mu} + k_1 r_{t-1} I^- + k_2 r_{t-1} I^+ - b_1 \pi_{t-1} - b_2 \pi_{t-2} - ... - b_{t-2}\pi_2 + \varepsilon_t \quad (23)$$

We can forecast the volatilities in the next period based on (23).

I divide the dataset into the "in-sample" estimation period and subsequent "out-sample" forecasting period. The estimation period contains 763 observations[15] and the forecasting period contains 100 observations. I use moving window to predict the volatility in the next period and show the predictions for the 100-day period in Figure 7. From Figure 7 we can see that ARFIMAX (0, d, 0, X) model did a good job in forecasting the future volatilities.

Besides the graph, I use two methods to measure the performance of the forecasting quantitatively. The first is mean square error, $MSE = \frac{1}{T}\sum(\ln \hat{h}_t - \ln h_t)^2$. The second method is to build a regression equation: $\ln h_t = \alpha + \beta \ln \hat{h}_t + \varepsilon_t$, if $\ln \hat{h}_t$ is the accurate forecasting of $\ln h_t$, we should have $\hat{\alpha} = 0, \hat{\beta} = 1$ and R^2 close to one. Table 4

[15] Daily volatilities generated using high frequency data. Each observation represents one daily realized volatility.

confirmed my result from the graph. As we can see in the table R^2 is greater than .40, and we cannot reject the hypothesis of $\hat{\alpha} = 0, \hat{\beta} = 1$ at 5% confidence level.

6. Conclusions

This paper first examines the properties of the realized volatilities of USD/CAD spot exchange rate over a three-year period using high-frequency intraday observations from Forxite. Most findings are consistent with previous studies. For example, this paper shows that the distributions of the standardized returns and the logarithmic realized volatilities are both approximately Gaussian, which is consistent with ABDL (2001a, 2001b). I find a unique heavy tailed M-shaped pattern for the average returns. This is because USD/CAD is rather a "locally" traded currency pair (mostly in North America) than a globally traded currency pair such as USD/JPY or USD/EURO. Although it has a unique look, the main pattern is still consistent with the theory. Using GPH test, I find a long-memory effect in the dataset. Because traditional ARCH models do not catch this effect, I use a fractionally-integrated model (ARFIMAX) to estimate the deseasonalized volatilities. This model catches the long-memory effect and the leverage effect in the dataset very well. The *MSE* is greater than $.6\$^2$ and the R^2 is greater than 40% for the measurement regression.

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Table 1

Daily Return Distributions

	Mean	*Std.Dev*	*Skewness*	*Kurtosis*	$Q(20)$	$Q^2(20)$
<u>Returns</u>						
USD/CAD	-.0001672	.0048404	.0585405	3.585104	28.8364 p=.0910	31.9799 * p= .0435
<u>Standardized Returns</u>						
USD/CAD	-.0002832	.0086389	.0350781	2.72022	23.9372 p= .2451	23.2346 p= .2774

Notes: * Significant at the 5% level. The top panel refers to the distribution of daily returns, while the bottom panel refers to the distribution of daily returns standardized by realized volatility. The columns labeled $Q(20)$ and $Q^2(20)$ contain Ljung-Box test statistics for up to twentieth order serial correlation in returns and squared returns, respectively.

Table 2

Daily Realized Volatility Distributions

	Mean	*Std.Dev*	*Skewness*	*Kurtosis*	$Q(20)$	*d*
<u>Volatility</u>						
USD/CAD	.5268207	.1378672	.8128098	5.38737	2175.8402*	
					p= .0329	
<u>Logarithmic Volatility</u>						
USD/CAD	-.675083	.2659235	-.4933379	5.458519	2120.5968*	.713709*
					p= .0304	

Notes: * Significant at the 5% level. The top panel refers to the distribution of realized volatility, while the bottom panel refers to the distribution of logarithmic realized volatility. The columns labeled $Q(20)$ contain Ljung-Box test statistics for up to twentieth order serial correlation in returns and squared returns, respectively.

Table 3

ARFIMAX Estimation

Model	*AR*	*d*	*MA*	k_1	k_2	AIC
(0, d, 0)		.267062*				1062.2
		(.02046)				
(1, d, 1)	.194936*	.480377*	-.560083*			1028.4
	(.07607)	(.02549)	(.06973)			
(0, d, 0, X)		.283165*		-28.1522*	.528804*	952.11
		(.01950)		(3.458)	(.06453)	
(0, d, 1, X)		.457085*	-.363081	-29.1278*	.567329*	957.67
		.04075)	(.06213)	(3.547)	(.06603)	
(1, d, 0, X)	-.218678	.368010*		-29.0640*	.553262*	967.73
	(.04053)	(.02715)		(3.514)	(.06552)	
(1, d, 1, X)	.191648	.480109*	-.555242	-28.9258*	.570530*	954.77
	(.07933)	(.02536)	(.07379)	(3.563)	(.06625)	

Notes: * Significant at the 5% level. Numbers in parenthesis are standard errors. Six alternative models are estimated and compared in this table. The last model is the model considering all the effects and it contains autoregression term, moving average term, and leverage effect term. Values of LLF and AIC are reported.

Table 4

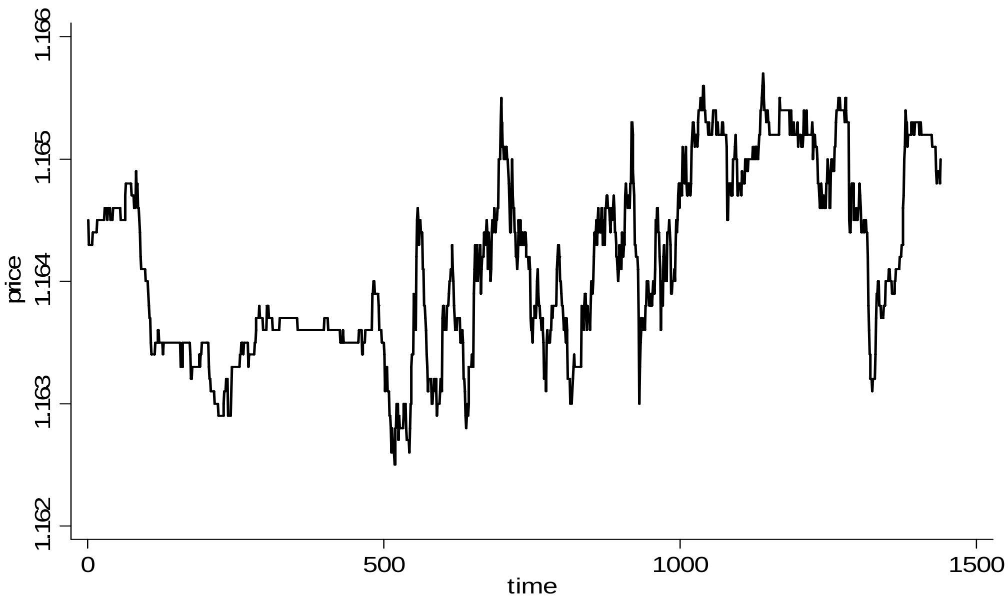
Forecasting Measurement

$\hat{\alpha}$	$\hat{\beta}$	R^2	MSE
-.6574 [-2.5373, 1.3268]	.90874 [.69120, 1.1238]	.4578	.64517

Notes: Numbers in brackets are the confidence intervals for the estimated coefficients. The coefficients mentioned in the second regression method are listed in the first three columns. MSE is reported in the last column.

Figure 1

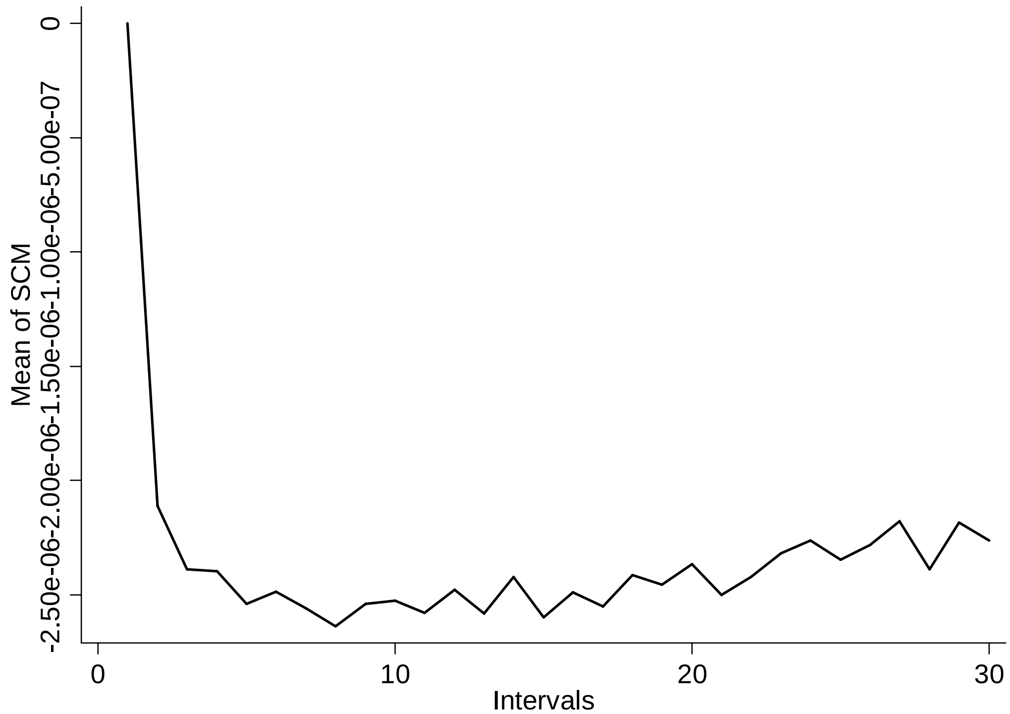
Five-minute Returns in Jan 02, 2007



Note: This figure is generated using the one-minute interval data in Jan 02, 2004. From the figure we can see that there are significant microstructure friction effects in the dataset.

Figure 2

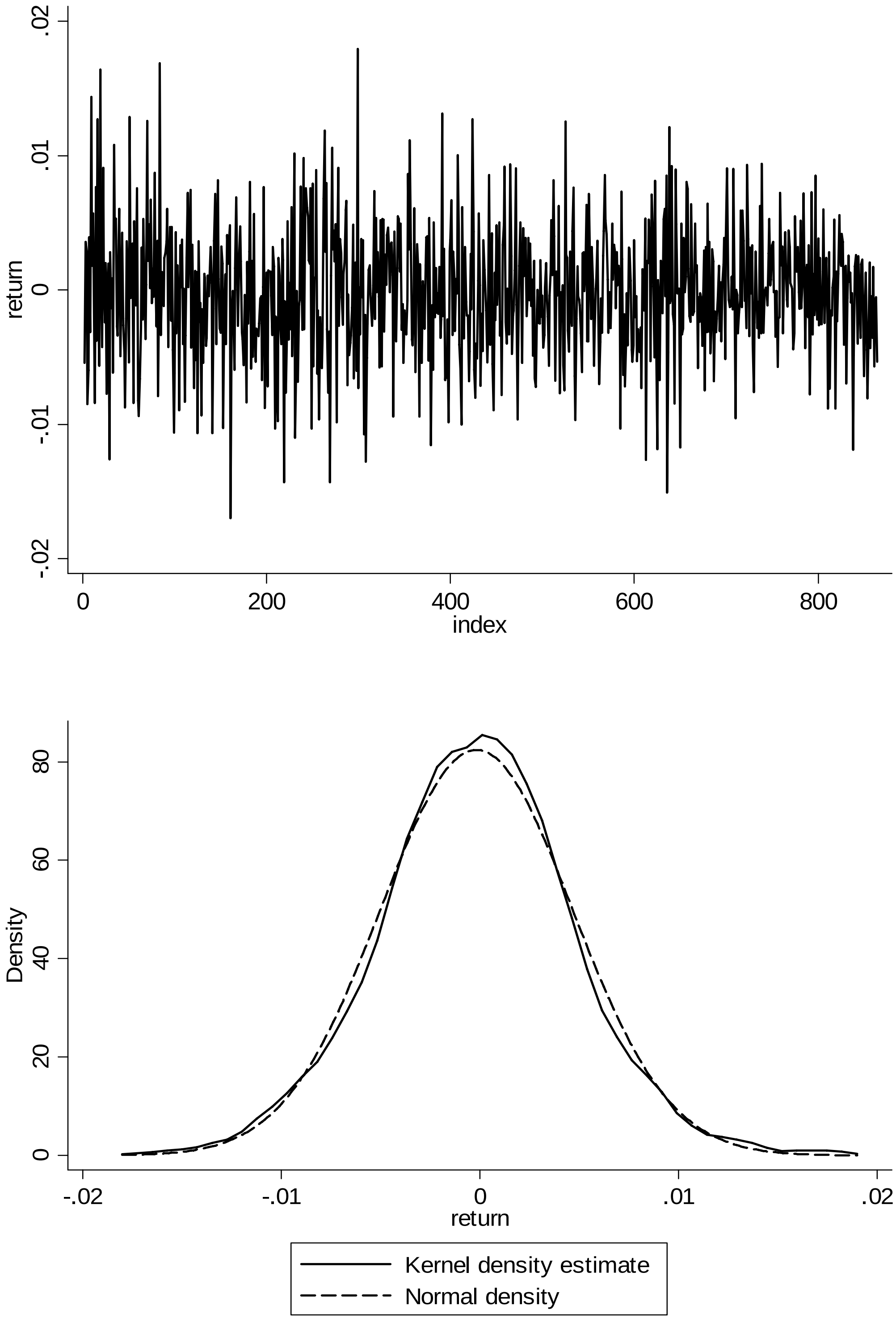
Plot of the Average SCM for Different Intervals



Note: Because dataset with too high frequency will have significant microstructure effects and the estimation with these microstructure effects will be noisy. Therefore before I estimate the model I need to either separate the microstructure effects from the dataset or select an interval which has a balance between high frequency and low microstructure. This figure shows the SCMs for different intervals. We can see from the figure that dataset with intervals between 10 and 20 has the lowest SCMs. In this paper, I use the dataset with fifteen-minute interval.

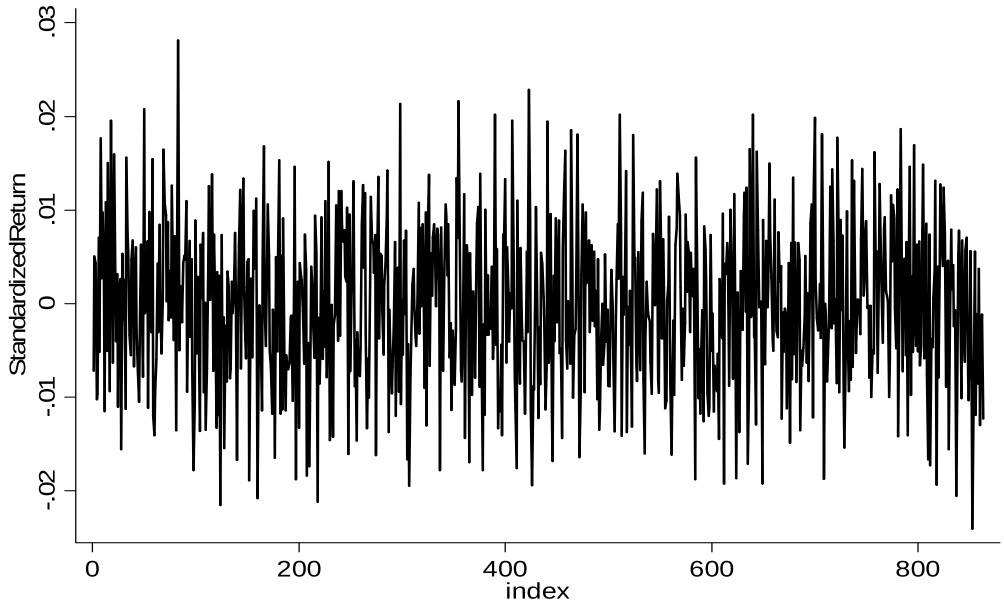
Figure 3(a)

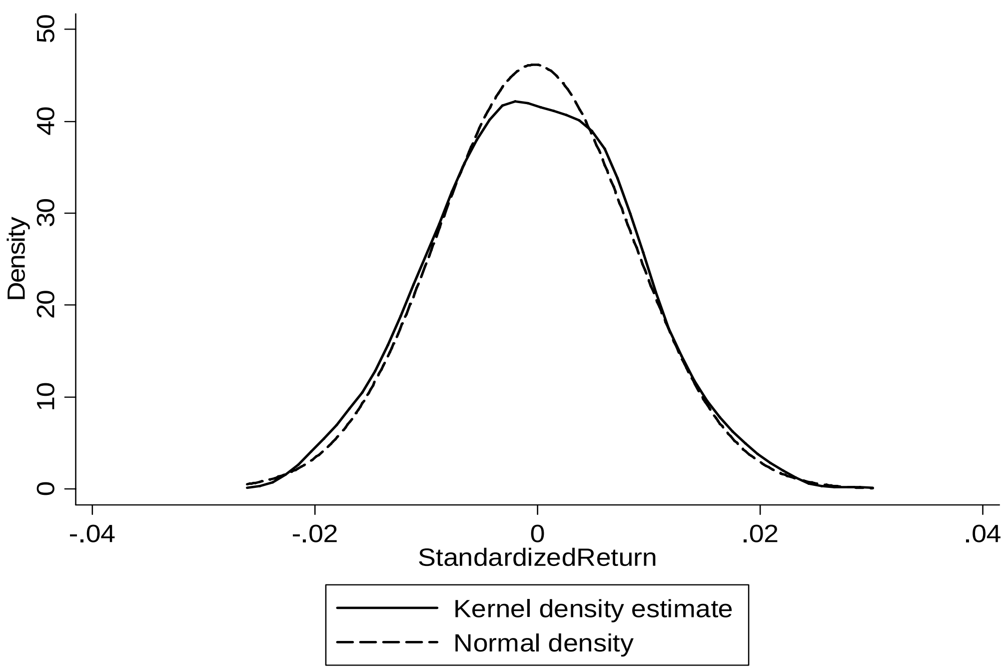
Kernel Estimates of the Density for Returns



Raw returns

Figure 3(b)



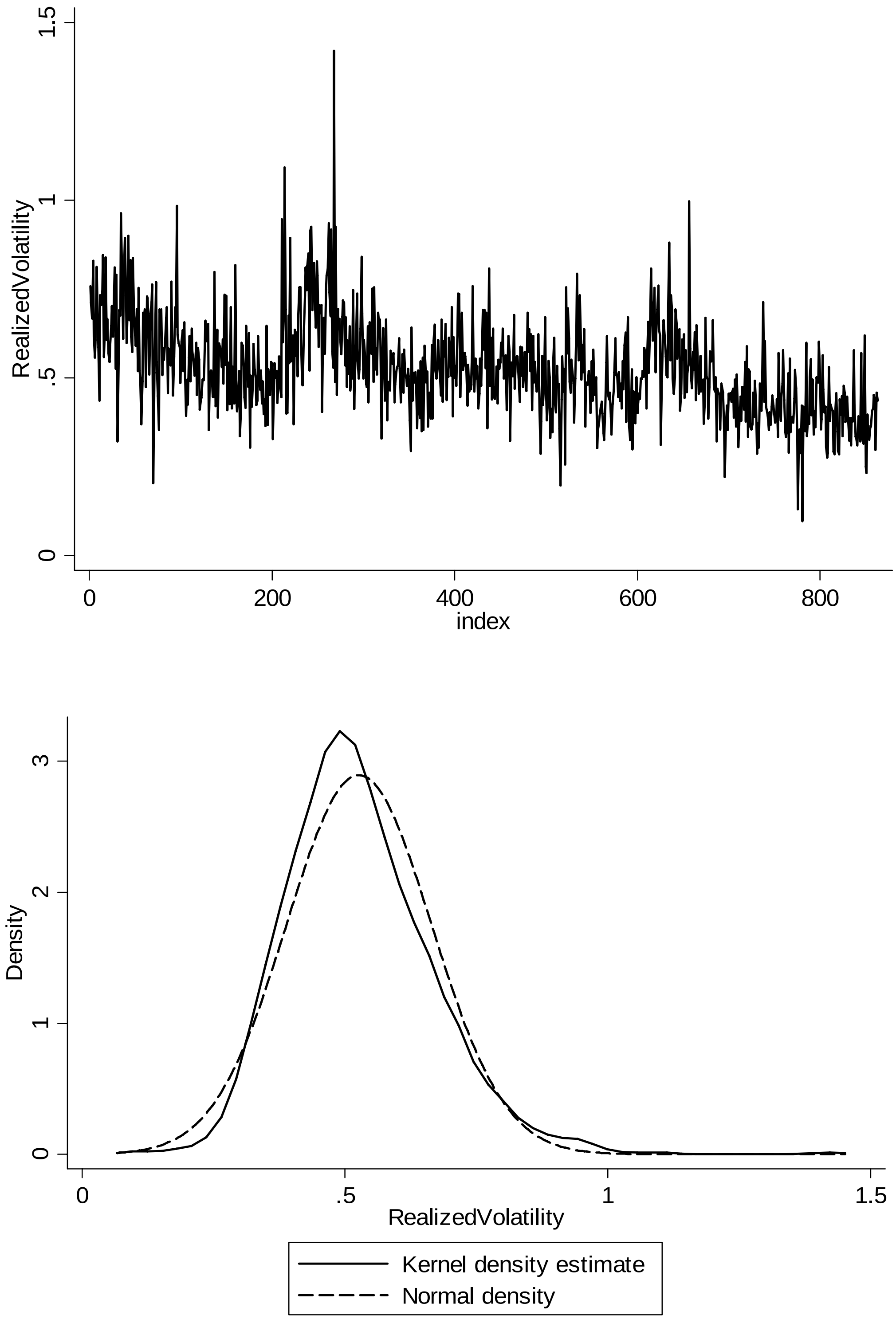


Standardized returns

Notes: I show kernel estimates of the density of daily returns on the exchange rate of USD/CAD. The sample period extends from January 2, 2004 through April 24, 2007. The solid line in figure 3(a) is the estimated density of raw returns. The solid line in figure 3(b) is the estimated density of returns standardized using its constant sample mean and time-varying realized standard deviation. The dashed lines in both figures are normal densities for visual reference.

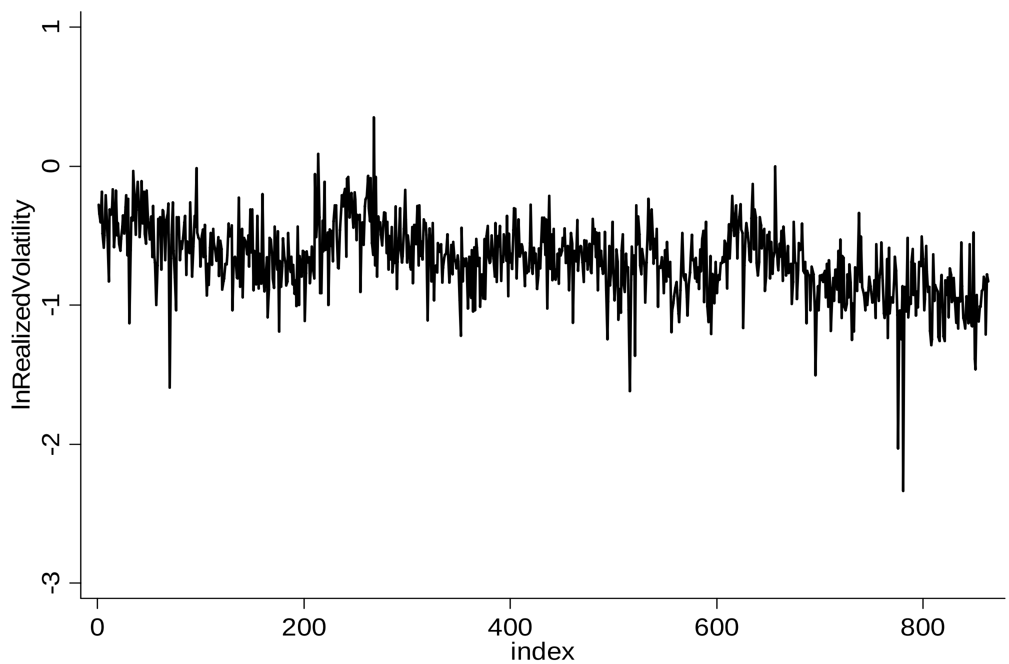
Figure 4(a)

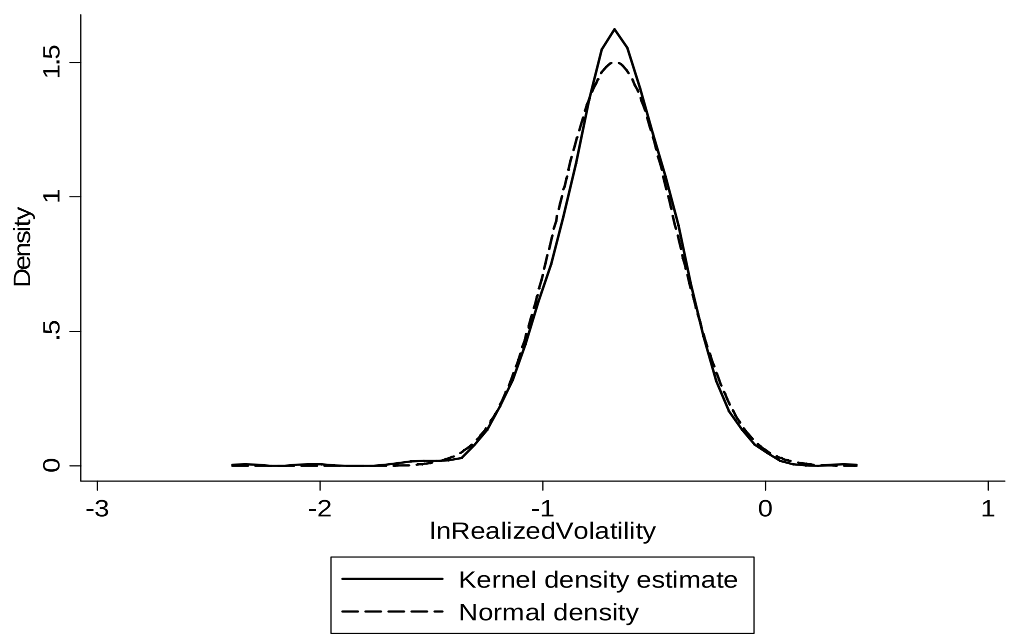
Kernel Estimates of the Density for Realized Volatilities



Realized Volatilities

Figure 4(b)



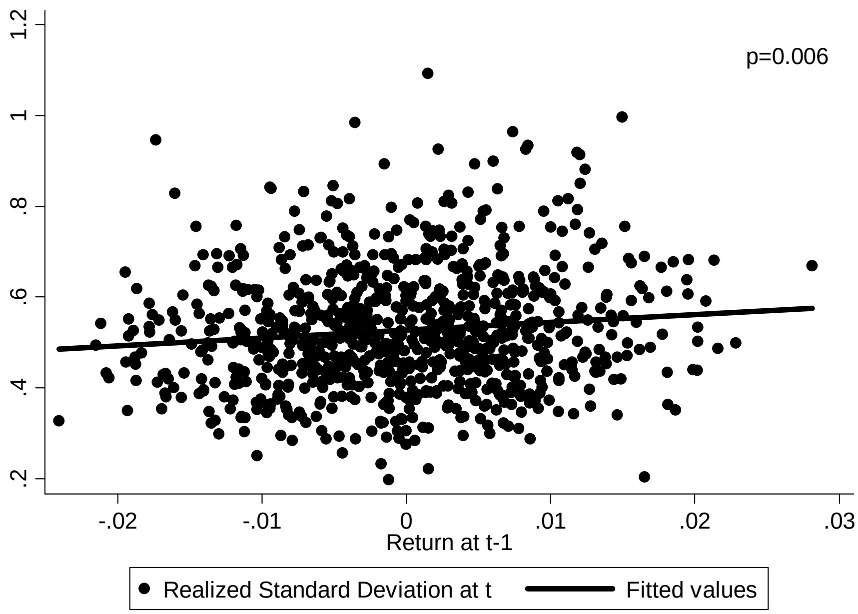


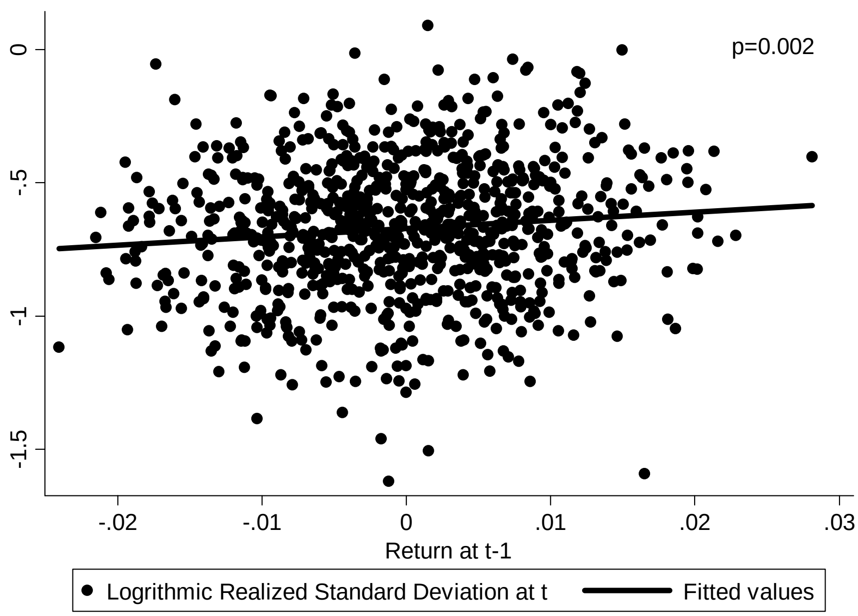
Logarithmic Realized Volatilities

Notes: I show kernel estimates of the density of daily realized USD/CAD volatility. The sample period extends from January 2, 2004 through April 24, 2007. The solid line in figure 4(a) is the estimated density of the realized standard deviation. The solid line in figure 4(b) is the estimated density of the logarithmic realized volatility. The dashed lines in both figure are normal densities for visual reference.

Figure 5

Leverage Effect

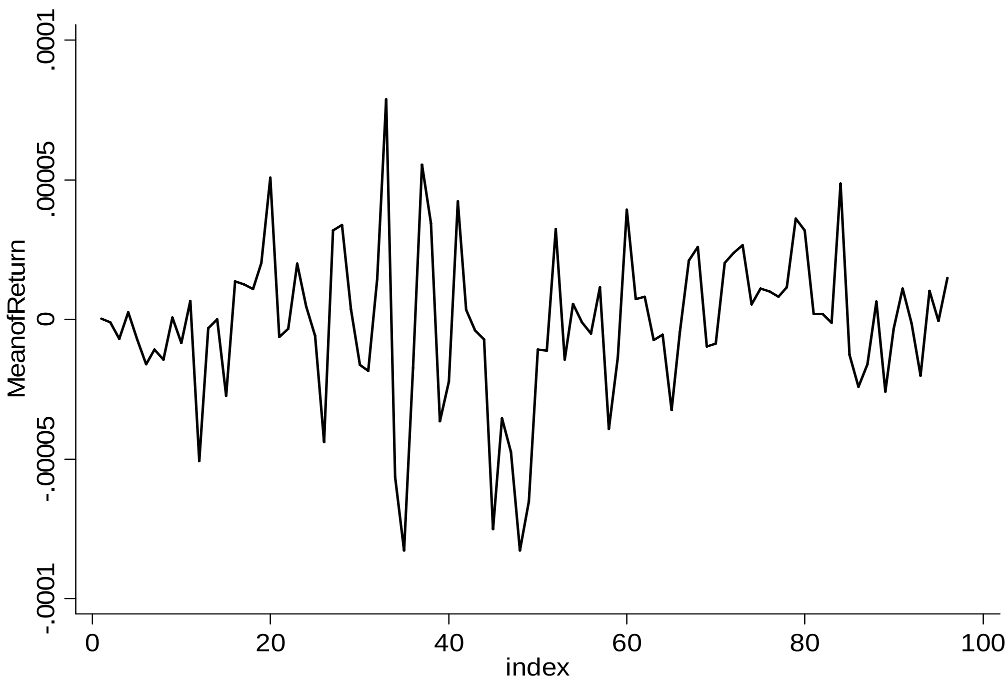




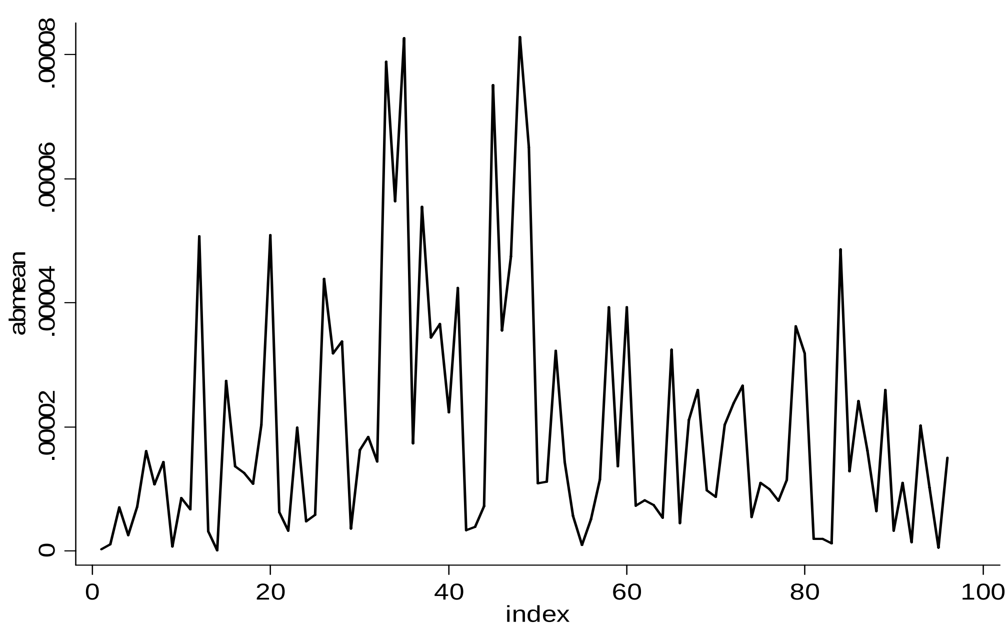
The graphs display lagged returns against standard deviation (top panel), and logarithmic standard deviation (bottom panel). The lines are OLS regression lines which are based on the displayed variable and a constant term. The regression *p*-values for the significance of intercepts are given in top-right corner of the plots.

Figure 6

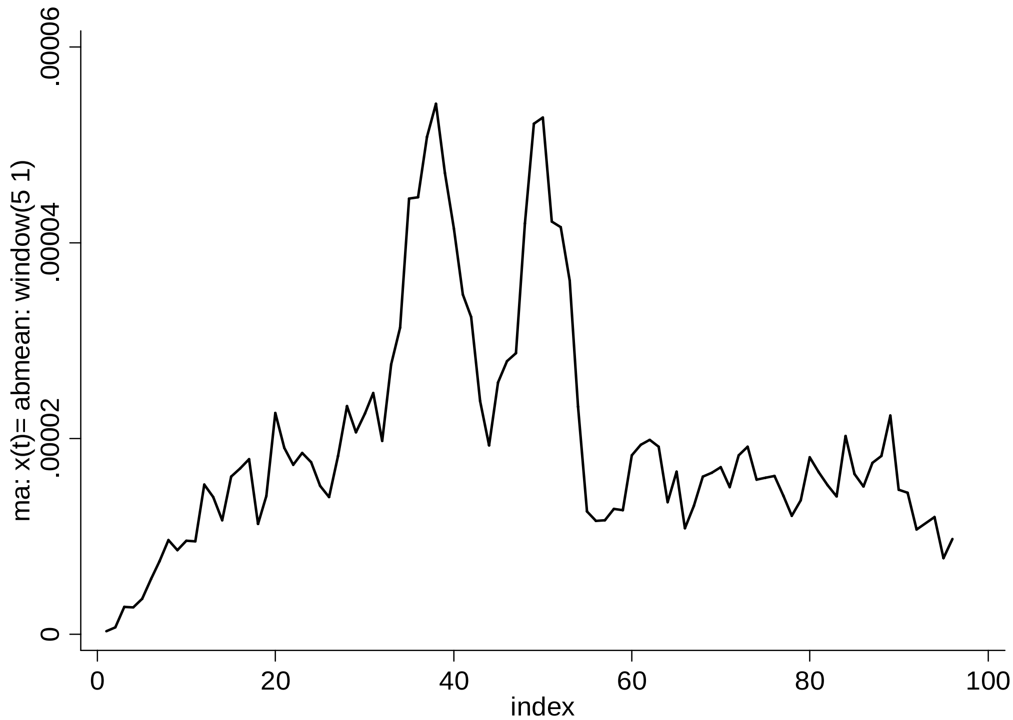
Intraday Periodicity



Average Returns



Absolute Value of Average Returns (Absolute Returns)

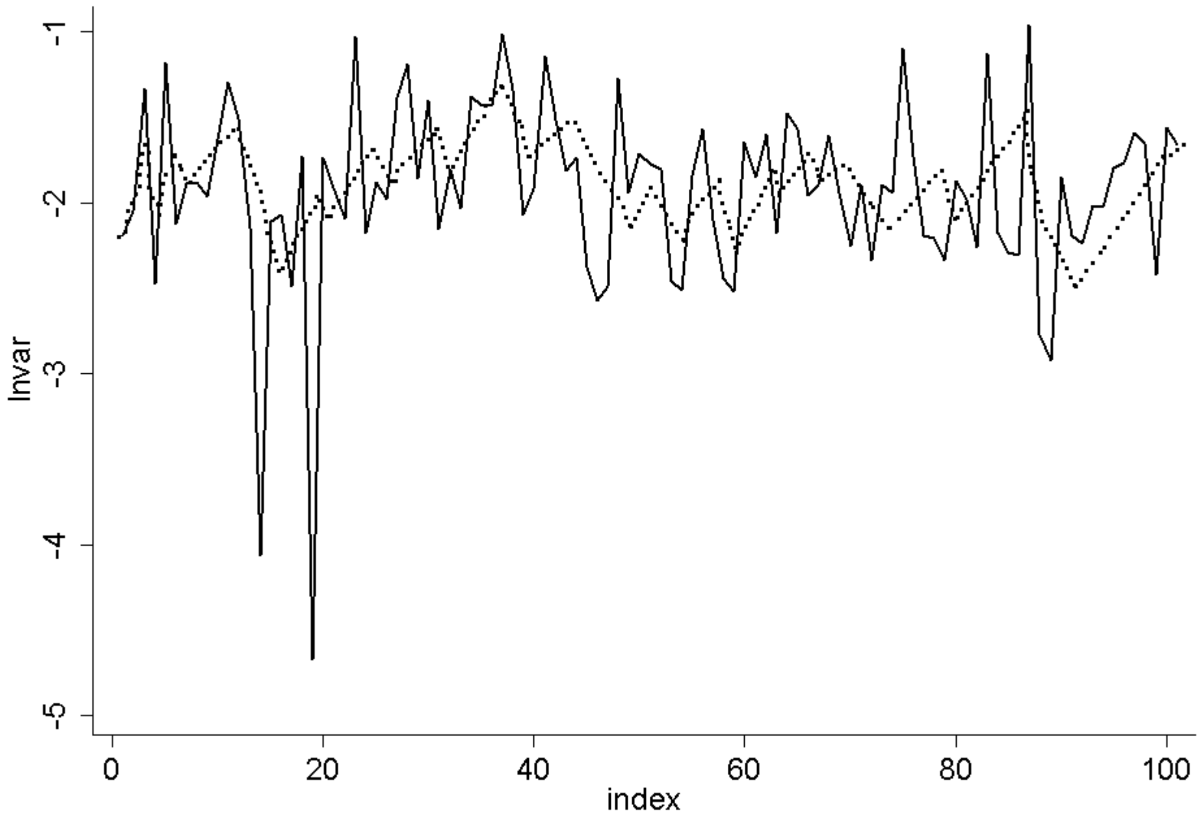


MA(5) of the Average Absolute Returns

Note: this figure plots the average returns from 0:00(EST) through 24:00(EST). We do not see a specific pattern in the first two graphs. However the third graph, the moving average of the absolute average returns, has a heavy tailed M-shaped pattern. See the main text for detail.

Figure 7

Forecasting using ARFIMAX (0, d, 0, X)



A TIME SERIES MODEL FOR PAIRS TRADING

1. Introduction

Statistical arbitrage has been a hot topic in both academia and Wall Street since the introduction of computational finance in early 80's. According to Prof. Andrew Lo, Statistical arbitrage "refers to highly technical short-term mean-reversion strategies involving large numbers of securities (hundreds to thousands, depending on the amount of risk capital), very short holding periods (measured in days to seconds), and substantial computational, trading, and IT infrastructure". "Pairs trading" is one of the statistical arbitrage strategies. This methodology was designed by a team of scientists from different areas (mathematics, computer sciences, physics, etc), which were brought together by the Wall Street quant Nunzio Tartaglia. The basic idea of pairs trading is to take advantage of market inefficiency: select a pair of stocks that move together in the history and trade them when they diverge by more than a pre-determined threshold. The idea is simple: if these two stocks move together in the history, they will converge back and the current disequilibrium (divergence) will be reset back to the equilibrium in the future. Profit will be made if this happens.

There are several other reasons for its popularity. First, since it does not normally evoke frequent intraday trading, pairs trading can be cost-feasibly automated. Second, it does not require cash flow and financial ratio based valuation models, which are potentially subjected to huge error margins. In pairs trading, valuations are relative and the position is often near market neutral. Lastly, it has sufficient flexibility to accommodate various investment styles such as pairs matched within sectors, size, index/non-index, growth and value, etc.

Although widely used by hedge funds and investment banks, pairs trading still remains elusive since it has not drawn nearly as much academic attention as contrarian trading. The latter involves ranking stocks based on past returns, then short sell leaders and buy followers to profit from short term overreaction. If prices systematically overreact, this implies positive expected profits from contrarian trading.

The academic research about pairs trading is still very young and most of the researches focus on three categories: naïve distance models introduced by Gatev et al (1999) and studied by Nath (2003) and Vidyamurthy (2004); cointegration models studied by Vidyamurthy (2004) and Herlemont (2006); and stochastic models by Elliot et al (2005), Do et al (2006) and Jurek and Yang (2006).

In both the constant threshold method and cointegration method, the underlying assumption is that the mean price distance between two parts of a pair (further in this paper referred to as "spread") and the distribution of this distance are constant over time. Although this may be valid in a short period of time, it is a relatively weak assumption and it cannot guarantee the trading strategy to be optimized all the time. Although there is no such assumption in stochastic models, most of these models use Autoregressive (AR) process to predict the mean (spread in pairs trading) and the predictability of these processes has been criticized by Donelson and Maltz (1972), Granger and Poon (2001), and Klüppelberg et al (2005). Therefore the trading performance based on the poor predictions is also questionable in stochastic pairs trading models.

This paper contributes to the literature by introducing a time series based trading strategy for pairs trading. This time series model forecasts the standard deviations of the spread series and uses the forecasted standard deviations as dynamic threshold values. This model removes the restriction of constant variance assumption in naïve distance models and adopts the GARCH model to overcome the low predictability of stochastic models. Another advantage of this time series model is that the background algorithm is simple and practical and this trading strategy can be easily embedded in most popular automatic trading platforms.

The remainder of the article is organized as follows. Section 2 provides some background on pairs trading strategy. The next section reviews three existing pairs trading models/methods. The model section describes my methodology of constructing a dynamic pairs trading strategy. The empirical results and strategy assessment are described in the next section, and the last section provides conclusions and directions for future research.

2. Background for Pairs Trading

2.1 Relative pricing

Asset pricing can be viewed in absolute and relative terms. Relative pricing means that the two assets that are close substitutes should be sold at same prices-it does not say what that price should be. In pairs trading, we use this relative concept since we are looking for the relative performances of the stocks without worrying about their absolute values. Therefore, pairs trading is a non-directional strategy in which the long and short positions offset the underlying exposure to fluctuations in the fundamental values of the two assets.

Relative distance[16] is used in pairs screening and the basic idea behind this method is that relative price difference between two assets is a measurement of co-movements between them. Gatev et al (1999) use sum of squared differences between two normalized prices series for pair screening. Prices are normalized because the original price series may have different means and the absolute *distance* is not comparable among series.

2.2 Strategy

The strategy is to implement long-short positions for the two stocks and make profit from the temporary misalignments. The starting assumption of this strategy is that stocks that have historically had the same trading patterns will do so in the future as well. If there is a deviation from the historical trend, this creates a trading opportunity, which can be exploited. Gains are earned when the price relationship is restored. More specifically, if the distance between two stocks' normalized prices is greater than a pre-set threshold value, the trader long the overvalued stock and short the undervalued stock. Under the previous assumption, when the two stocks converge, the trader closes the trade and makes profit.

Fortunately the above characteristics can be caught and modeled by a mean-reverting process: if the spread between two stock prices follows a mean-reverting process, the deviation of the spread from its long-run mean (i.e. zero) is a sign of

[16] Based on Perlin (2007), using correlation criteria gives the similar result in pairs screening.

mispricing and long-short position should be executed and profit will be made when the spread reverts to its mean.

A Dickey-Fuller test (Dickey and Fuller (1979)) can determine the stationarity of the spread $P_t^A - P_t^B$ as follows:

$$\Delta(P_t^A - P_t^B) = \mu + \gamma(P_{t-1}^A - P_{t-1}^B) + \varepsilon_t \qquad (1)$$

where P_t^A is the price of stock A at time *t* and P_t^B is the price of stock B at time *t*, and the null hypothesis is $\gamma = 0$, meaning the spread is not mean reverting. If the null hypothesis can be rejected on the 99% confidence level the spread of stock prices follows a weak stationary process and is therefore mean-reverting. According to Herlemont (2006) if the confidence level is relaxed, the pairs do not mean-revert good enough to generate satisfactory returns.

3. Existing Pairs Trading Methods

3.1 The constant threshold method

This method is straight forward and it is used by a lot of investors due to its simplicity. Gatev et al (2006) use this method in their paper. In their paper they first select the pairs and then use a pre-specified threshold (two standard deviations) as the trigger of a trade. The trading position opens when spread between the total return indices of two securities diverges by "two historical standard deviations, as estimated during the pairs formation period". When the spread is less than two standard deviations the investor closes the pairs trading. In their paper, they work with daily stock data over 1962-2002 and the top pairs selected using the above simple rule generate annualized excess returns[17] of up to 11%. Nath (2003) applies pairs trading strategy to the entire universe of securities in the highly liquid secondary market for U.S. government debt and compares the performances of this simple strategy with four different open and close thresholds. This paper is unique compared to other pairs trading studies because the database used in this study is intraday data rather than daily data and the whole dataset has 4.5 million trades and approximately 50 million quotes for 829 securities over 1994-2000. He concludes that a simple pairs trading strategy with 15th percentile as the open trigger and 5th percentile as the close trigger is preferred for U.S treasury securities.

Vidyamurthy (2004) calculate an optimal threshold in the case where the spread is Gaussian white noise series. His approach is as follows: based on a constant threshold method, the investor buys one unit of the spread whenever he observes that the spread has a value less than or equal to the negative of, the predetermined constant threshold ($-\Delta$). Similarly, he sells one unit of the spread when he observes a value greater than or equal to Δ. Since the spread series are assumed to be Gaussian white noise, the probability that this series at any time point deviate by amount greater than or equal to Δ is determined by the integral of the Gaussian process, equal to 1–N(Δ), where N(Δ) is the integral $\int_{-\infty}^{\Delta} f(x)dx$. Assume the investor trades in T time steps and he can expect to have T instances greater than Δ. Similarly, the probability of the value being less than or equal to $-\Delta$ is given by N($-\Delta$). Since Gaussian series are symmetric we have N($-\Delta$) = 1 – N(Δ) and therefore the number of instances, we expect the value of the spread to be less than or equal to $-\Delta$ is also T(1–N(Δ)). Thus, in a time span of T units the investor can expect to have bought and sold the spread an average of T times. And the profit on each

[17] The definition of excess return in pairs trading is different from traditional definition (returns in excess of the risk-free rate). This will be explained in section 5.

round turn (buy and sell) is 2Δ. Profit in time period T is calculated using Profit = profit per trade × number of trades, and in this case it is $2T\Delta(1-N(\Delta))^2$. The optimal threshold can be calculated based on maximizing this profit function.

While it is hard to calculate the extreme value by taking the first derivative, profit plot is much easier and can give us an approximate result. For profit plot, see Vidyamurthy (2004). According to Vidyamurthy, the approximate threshold that maximized the above profit function is 0.75σ.

Although the constant threshold method is straight forward and easy to use, there are several pitfalls in this method according to Jurek and Yang (2006). These risks are present in essentially all relative value trades and include the uncertainty about the timing at which the mispricing will be eliminated (After trade is open, when to close the trade is also important. The uncertainty of the timing to close the trade is usually called horizon risk) and the potential for the mispricing to diverge far from its mean prior to convergence (it is possible that the two stocks continue to diverge from each other after trade is open and this risk is usually called divergence risk). These two risks make this method very hard to be applied in practice. Another problem with this method is that it is non parametric and therefore it does not have any predicting power.

3.2 The cointegration method

Vidyamurthy (2004) introduced a cointegration approach for pairs trading using the co-integration theory proposed by Engle and Granger (1987). The cointegration theory says that each element of a vector of time series x_i, first achieves stationarity after differencing, but a linear combination $\alpha' x_i$, is already stationary, the time series x_i, are said to be co-integrated with co-integrating vector α . The simplest case of cointegration is two time series that are both integrated of order 1, can be linearly combined to produce a new single time series that is integrated of order zero or stationary.

Cointegrated time series can also be represented in Error Correction Model (ECM) in which the movement of current period is correlated with the correction of last period's deviation from the equilibrium. According to Vidyamurthy (2004), the logarithmic stock prices are often assumed to be random walk and there is a good chance that they will be cointegrated. If that is the case, cointegration result can be used to determine how far the spread is away from its equilibrium and this can be used as a trigger for trading pairs.

Vidyamurthy (2004) adopts Engle and Granger's approach to test cointegration. This is conducted in two steps: first log price of stock A is regressed against log price of stock B:

$$\log(P_t^A) - \gamma \log(P_t^B) = \mu + \varepsilon_t \tag{2}$$

where γ is the cointegration coefficient and the constant μ captures some sense of "premium" of stock A over stock B.

Second, the residual calculated from above equation is tested for stationarity using Augmented Dickey-Fuller test.

Error Correction Model is a step toward determining how the variables are linked together after the cointegration test. If the residual is tested to be stationary with Engle and Granger's approach, If cointegration is supported by (2), the parameters of an ECM can be estimated and give more information on how the variables are related. Herlemont (2006) gives the following estimation equations:

$$\Delta\log(P_t^A) = \alpha_1 + \alpha_A(\log(P_{t-1}^A) - \phi\log(P_{t-1}^B) + \sum_{i=1}^{p} a_{11}^{(i)}\log(P_{t-i}^A) + \sum_{i=1}^{p} a_{12}^{(i)}\log(P_{t-i}^B) + \varepsilon_{At} \quad (3)$$

$$\Delta\log(P_t^B) = \alpha_2 + \alpha_B(\log(P_{t-1}^A) - \phi\log(P_{t-1}^B) + \sum_{i=1}^{p} a_{21}^{(i)}\log(P_{t-i}^A) + \sum_{i=1}^{p} a_{22}^{(i)}\log(P_{t-i}^B) + \varepsilon_{Bt}$$

Note that in the first step if log price of stock B is regressed against log price of stock A (remember log price of stock A is regressed against log price of stock B in equation (2)), the residual test in the second step will be different and therefore the ECM will be different. Although this issue can be resolved by using the t-statistics from Engle and Yoo (1987), this model is complicated compared to other models. Another issue in this cointegration method is that if the bivariate series are not cointegrated, the "cointegrating regression" leads to spurious estimators (Lim and Martin, 1995) and make the mean reversion analysis unreliable.

3.3 The stochastic method

Stochastic pairs trading models study the level of mispricing and the strength or timing of mean-reverting process. And based on these, the investor determines the tradability of the spread and makes entry and exit decisions.

Elliot et al (2005) proposed a stochastic method and tried to model the spread between two assets using an Ornstein-Uhlenbeck (OU) process.

The spread is modeled as follows:

$$dS_t = k(\bar{S} - S_t)dt + \sigma dZ \quad (4)$$

where B_t is a risk free asset with a discount rate of r. S_t is the spread following a mean-reverting process and $\bar{S}$ is its long-run mean. S_t is known to converge to $\bar{S}$ at a speed of k . dZ is a standard Brownian motion in a predefined probability space. This equation simply says the next change in the spread is opposite in sign to the deviation of the spread from its long-term mean, with a magnitude that is proportional to the deviation. When $S_t > \bar{S}$, the investor shorts the spread asset (long the undervalued security and short the overvalued security) and invests the proceeds in the risk free asset. The strategy is reversed when $S_t < \bar{S}$.

Compared to the constant threshold model, this stochastic model offers two major advantages. First, spread is modeled with an mean-reverting OU process, and this process is appropriate since it catches the horizon risk by modeling the uncertainty over the length of the time that will elapse before the process converges to its long-run mean and catches the divergence risk by modeling the variance distribution of the spread between its current value and its first reversion to the long-run mean.

Second, it is parametric and the parameters can be estimated and be used to predict future values. The estimator is a maximum likelihood estimator and optimal in the sense of minimum mean square error (MMSE).

The disadvantage of these stochastic models is that they have relatively low predictability in mean prediction. This is understandable since OU process is basically an autoregressive process and its simple form does not catch much information about the mean. And for this reason, the stochastic method is rarely used in practice.

4. A New Pairs Trading Model: The Time Series Pairs Trading Model

My approach is to introduce time series models in pairs trading and take advantage of the consistent predictability of variances in time series models. This approach is conducted in three steps: first the optimal threshold is calculated based on the steady state where price distance and distribution of spread are constant. This optimal threshold yields the highest profit and is a function of the variance. Second, the time series characteristics of the spread (or return spread) are examined and an appropriate time series model is used to predict the future variances of the spread (or return spread). Third, a dynamic threshold, calculated based on the optimal threshold function and the predicted variance, is used as a dynamic trading trigger.

4.1 Time Series Characteristics of the Spread

Data and Pairs Selection

My analysis focuses on the stocks traded in the United States. The raw dataset from The Center for Research in Security Prices (CRSP) consists of daily closing prices for 12,895 stocks traded in the major US stock exchanges during the sample period, January 1, 2000 through April 30, 2008. Using Getav et al's screening method, I screen out all stocks with one or more days with no trade. This serves to identify relatively liquid stocks and facilitate pairs formation. The screened dataset consists of daily close prices for 3091 stocks. Each stock has 2093 observations. I use the first 1839 observations as the initial training. The remaining 254 observations starting from April 30, 2007 to April 30, 2008 represents the effective trading days in one year.

As I mentioned in the previous section, all the prices are normalized since different stocks have different means and the absolute distance among them is meaningless in my research. After normalization, all the stocks are brought to the same mean and this permits formation of pairs.

The normalization is based on the following equation:

$$P_{it}^{N} = \frac{P_{it} - E(P_{it})}{\sigma_i} \quad (5)$$

where P_{it}^{N} is the normalized price for stock i at time t, $E(P_{it})$ is the expected value of that stock and it is the mean in this case and σ_i is the standard deviation of this stock.

The next step is to choose, for each stock, a pair that has minimum absolute distance between the normalized prices. Again I use the approach introduced in Gate et al (1999) where a matching partner for each stock is chosen by finding the stock that minimizes the sum of squared deviations between the two normalized price series[18].

After the pairs selection, I study the performances of top 5 and top 20 pairs with the smallest historical distance measure.

GARCH model

Traditional econometric models assume a constant one-period forecast variance. To relax this implausible assumption, Engle (1982) developed a class of models called autoregressive conditional heteroscedasticity (ARCH). These are zero mean, serially uncorrelated processes with nonconstant variance conditional on the past. In this paper, I use ARCH class model to model and forecast the nonconstant variance and use that to build a dynamic optimal threshold.

A useful generalization of ARCH model is the GARCH parameterization introduced by Bollerslev (1986). This model is also a weighted average of past squared

[18] The MatLab code for pairs selection was provided by Perlin on www.mathworks.com.

residuals, but it has declining weights that never go completely to zero. Below is the original GARCH model:

$$y_t = x_t{}' \beta + \varepsilon_t \tag{6}$$

$$\varepsilon_t = \sqrt{h_t} \cdot v_t \tag{7}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \theta_1 h_{t-1} + \cdots + \theta_p h_{t-p} \tag{8}$$

$E(v_t) = 0, D(v_t) = 1$, $E(v_t v_s) = 0 (t \neq s); \alpha_0 > 0, \alpha_i \geq 0, \theta_j \geq 0, \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \theta_j < 1$

The above process is called GARCH(p,q) process. In the third equation $h_t = \mathrm{var}(\varepsilon_t | \varphi_{t-1})$, φ_{t-1} it is the information before time t-1.

Because GARCH(p,q) is an extension of ARCH model, it has all the characteristics of the original ARCH model. And because in GARCH model the conditional variance is not only the linear function of the square of the lagged residuals, it is also a linear function of the lagged conditional variances, GARCH model is more accurate than the original ARCH model and it is easier to calculate.

The most widely used GARCH model is GARCH(1,1) model. The (1,1) in parentheses is a standard notation in which the first number refers to how many autoregressive lags, or ARCH terms, appear in the equation, while the second number refers to how many moving average lags are specified, which here is often called the number of GARCH terms. Sometimes models with more than one lag are needed to find good variance forecasts. GARCH(1,1) is the most widely used GARCH model because of its accuracy and simplicity. The GARCH(1,1) model looks like this:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \theta_1 h_{t-1} \tag{9}$$

where α_0 is the constant, α_1 is the coefficient for first order ARCH effect (autoregressive lags), and θ_1 is the first order GARCH effect (moving average lags). According to the assumptions in equation (8), this model requires all the coefficients to be positive.

GARCH Characteristics of the Spread

In this part, I study the time series characteristics of the spread with GARCH(1,1) model using the first 1839 observations. In this part, I define a spread as the difference between the normalized prices of the two stocks:

$$SP_{ijt} = NP_{it} - NP_{jt} \tag{10}$$

where SP_{ijt} is the spread between stock *i* and stock *j* at time *t*, NP_{it} is the normalized price for stock *i* at time *t*, and NP_{jt} is the normalized price for stock *j* at time *t*.

The summary statistics for top 20 pairs are shown in Table 5. As expected, the average of the means of these top 20 pairs is 1.108E-10, which is close to zero. This is because all the spreads are mean-reverting[19] and they fluctuate around their long-run mean of zero. From the daily standard deviation, the spread between AMB Property Corporation (AMB) and ProLogis (PLD) is the most volatile and the spread between Developers Diversified Realty Corp (DDR) and Macerich Co (MAC) is the least volatile. The mean skewness is 0.231356 with the maximum of 1.749457 and the minimum of -0.5686995. Among all the pairs seven are negatively skewed and thirteen are positively skewed. The mean value of the kurtosis is 3.467160 with the maximum of 6.506454 and

[19] According to the Dickey-Fuller test addressed in section 2.2, null hypothesis that the series is not mean-reverting is rejected at 1% confidence level for all 20 pairs.

the minimum of 2.427899. Seven pairs have kurtosis greater than three, which is the normal value, and therefore these pairs show evidence of fat tails.

The patterns of the spreads of top 4 pairs are plotted in Figure 8and Figure 9. Miller (1979) mentioned that the residuals of a fitted model seem to be autocorrelated. Therefore, it seems reasonable to consider the volatilities (or variabilities) of the spreads for these pairs. Since the means for all pairs are close to zero, we consider the autocorrelation function plot of SP_{ijt} for each pair and these plots are shown in Figure 4.1. It shows that there is a substantial dependence among spreads for each pair. Therefore it is appropriate to use GARCH model to model the residuals.

The results of the GARCH (1,1) estimations are shown in Table 6. The three coefficients in the variance equation (9) for each pair are listed as α_0, α_1 and θ_1. All pairs have significant ARCH effect and GARCH effect except for the pair of Essex Property Trust and Boston Properties, and the pair of Essex Property Trust and BRE Properties. Notice that the coefficients for each pair sum up to a number less than one, which is required to have a mean reverting variance process. Since the sums for all pairs are very close to one, these processes only mean revert slowly.

The estimation is conducted using the sample from January 01, 2000 to April 29, 2007, which has 1839 observations. The conditional standard deviations[20] h_t for the out of sample observations, which is from April 30, 2007 to April 30, 2008, are calculated recursively using the estimated variance equation. Figure 9 shows the time series plot for

[20] Because standard deviation is used in constant threshold method and to keep consistent I use standard deviation instead of variance in my time series model.

the predicted conditional standard deviations of the out of sample observations for the top 4 pairs.

4.2 Dynamic Threshold Method

Dynamic threshold method is a modification of the constant threshold method used in Gatev et al (2006), Nath (2003), Vidyamurthy (2004) and Perlin (2007). Recall in constant threshold method, the trading is triggered when the normalized prices diverge by more than 0.75, or 2 in Gatev et al, of the historical standard deviation of that pair. This threshold value is constant across the whole trading period since the historical standard deviation obtained during the pairs formation period does not vary. In dynamic threshold method, instead of using a constant standard deviation, I use the predicted standard deviations generated from GARCH model. Compared to a constant standard deviation, this predicted value calculated using the moving window[21] can catch the evolution of the prices and make the trading strategy more dynamic.

Recall in Section 4.2.3, for each trading day I calculate a particular conditional standard deviation based on the estimated GARCH model and previous information. The divergence of the pair prices in each day is thus compared with 0.75 of the predicted conditional standard deviation in that day. I open a position in that pair when the prices have diverged more than that particular threshold value. This particular trigger value is used during that trading interval until the prices have reverted and thus the position is closed. After the position is closed, 0.75 of the predicted dynamic standard deviations are again used as dynamic threshold values until the position open next time.

[21] This window contains 1839 previous observations for each prediction.

The reason why I use a fixed threshold value instead of using the predicted dynamic threshold values after the trade is open and during the trading interval is nontrivial in my method. Pairs that open and converge during the trading interval will have cash flows. In constant threshold method, all the cash flows are guaranteed to be positive since the distance between the two stocks are guaranteed to be closer at the end of the trading interval than at the beginning of the interval. While in dynamic threshold method, if the next threshold value is larger than the previous one and it triggers the closing of the position, a negative cash flow is generated. Let us check one simple case to see the risk of using dynamic threshold values and this can be examined in details from Figure 10. In Figure 10 the position is opened at day one when the pair prices have diverged more than the threshold value on that day, which is calculated as 0.75 of the particular standard deviation in that day. On day two, I have a predicted standard deviation larger than that on day one, and the position is closed because the distance between the prices is less than 0.75 of this predicted standard deviation. In this case, a negative cash flow is generated and this is definitely an unattractive trading strategy for investors. Therefore, using a fixed threshold value during a trading interval will avoid this negative return problem and guarantee positive cash flows assuming they converge.

5. Assessing Performances Based on Different Trading Strategies

5.1 Excess Return Computation

In practice, the return or profit is calculated in the following way: if the position opens and converges during the trading period, there is a positive cash flow and if this process repeats within the trading period there will be a series of positive cash flows; if the position opens and never converges during the trading period, the position is closed at the end of the period no matter the return is positive or negative. Therefore during a particular trading period, there will be zero, one or more than one positive cash flows during the period and a positive or negative cash flow at the end of the period. Because the gains and losses of trading are computed over long-short positions of one dollar, the payoffs have the interpretation of excess returns. According to Perlin (2007), the general equation to calculate the excess return is as follows:

$$R_E = \sum_{i=1}^{n}[\sum_{t=1}^{T} R_{it} I_{it}^{L\&S}] + (\sum_{i=1}^{n}[\sum_{t=1}^{T} Tc_{it}])[\ln(\frac{1-C}{1+C})] \quad (11)$$

where R_{it} is the real return of stock *i* at time *t*, calculated by $\ln(p_{it} / p_{i(t-1)})$. $I_{it}^{L\&S}$ is a dummy variable that takes value -1 when stock *i* is the leader and a long position is created for it at time *t*, value 1 when stock *i* is a follower and a short position is created and 0 otherwise. Tc_{it} is a dummy variable that takes value 1 if a transaction is made for asset *i* at time t and 0 otherwise. For each trading interval the transaction cost is only counted once since during the interval the stocks are held instead of traded by the investor. *C* is the transaction cost per transaction and it is calculated as a percentage of each trade (I use *C*=0.1% in this paper). *T* is the number of effective trading days and it equals 254 in this paper.

After the returns for each stock are calculated, the total return for that pair is calculated by summing up the returns of the stocks that comprise the pair[22]. The excess returns are calculated based on the rule that all trades are executed at the end of the day when the threshold comparisons were conducted.

[22] Equation (11) gives the general form of return computation. n is not limited to be two where the return for one pair is calculated. This general equation can be used to calculate the return for a portfolio where there are two or more pairs.

5.2 Optimal Threshold Function

According to Vidyamurthy (2004), with the assumption that the spread follows Gaussian white noise process, the threshold that yields the highest profit is 0.75σ. Vidyamurthy also examines the case where the inventory is restricted to be one spread unit at each time. In pairs trading, inventory is defined as the average trade volume of the two stocks comprising the pair. Based on this restriction the investor buys one unit of the under priced stock and sells one unit of the over priced stock when the spread is more than the predetermined threshold. Vidyamurthy ran a simulation using 5,000 white noise realizations and concluded the result still hold with this restriction.

Vidyamurthy proves, in theory, 0.75σ is the threshold function that yields the highest profit, but he does not perform empirical analysis using real data in his book. In this paper, I compare the returns of my top pairs based on the thresholds of 0.75σ and 2σ [23] respectively and the results are addressed in Table 7. Hypothesis testing for comparing the mean returns using two different threshold values are tested using paired t-test. The results suggest that the mean returns using 0.75σ are significantly higher than those using 2σ for both top 5 pairs and top 20 pairs at 10% significance level. Therefore in this paper, I use 0.75σ as the optimal threshold function in both traditional constant threshold method and my new dynamic threshold method.

5.3 Trading Period

[23] 2σ is widely used in most naïve pairs trading models such as Nath (2003), Herlemont (2006), Getav (2006) and Perlin (2007).

In this section, I compare the performance of dynamic versus constant threshold methods for top 5 and top 20 pairs. The trading period is one year (April 30, 2007 to April 30, 2008) and the first trading day is the day following the last day of pairs selection period. Figure 11 and 5.2 illustrate the pairs trading strategy for two stocks, Avalon Bay Communities and Boston Properties, in the three-month period starting from July 30, 2007 to October 30, 2007 based on constant threshold method and dynamic threshold method respectively.

The top panel (panel A) in each figure shows the normalized prices of the two stocks with dividends reinvested. This pair is the 12th on the list of the top 20 pairs and we can see the co-movement of these two stocks is significant during this period. Panel B in Figure 11 shows the threshold value of the constant threshold method. This value is calculated as 0.75 of the historical standard deviation which is obtained during the pairs formation period. This value is constant over the whole trading period. Panel B in Figure 12 shows the dynamic threshold values. These values are calculated using the GARCH model we discussed in Section 4.2.3 and the trading strategy is implemented based on the rule defined in Section 4.3. As we can see in this panel, after each position is opened, the threshold values are fixed at the level where the trade is first triggered in that trading interval. That is where those platforms[24] in that panel come from. The bottom panel (panel C) in each figure shows the trading positions during this trading period. The kinked lines indicate the opening and closing of the strategy on a daily basis.

5.4 Strategy profits

[24] Recall we do not see flats in figure 4.2, which shows the predicted standard deviation.

The excess returns for different trading methods are summarized in Table 8 and Table 9. Panel A in Table 8 shows the excess return distribution for top 5 and top 20 pairs using dynamic threshold method. The average annual excess return is 21.3% for top 5 pairs and 7.8% for top 20 pairs[25]. Panel B shows the excess return distribution for top 5 and top 20 pairs using constant threshold method. The average annual excess return is 18.9% for top 5 pairs and 6.2% for top 20 pairs. These excess returns are large in economical and statistical sense, and suggest both pairs trading methods are profitable. Besides the average excess returns, Panel A and Panel B of Table 8 also provide information about the excess return distributions. And we can see that dynamic threshold method has smaller standard deviations for excess returns for top 5 pairs and top 20 pairs.

Table 9 shows the returns for each pair using the two methods. In Panel C of Table 9, the relative performances for two methods are summarized. For top 5 pairs, four out of five pairs earned higher excess returns with dynamic threshold method than with constant threshold method. For top 20 pairs, thirteen out of twenty earned higher excess returns with dynamic threshold method. Hypothesis testing for comparing the mean returns of the two methods are tested using paired t-test. The results suggest that the mean returns for dynamic threshold method are significantly higher than those of the constant threshold method for both top 5 pairs and top 20 pairs at 10% significance level.

6. Conclusions and Future Research

This is the first paper to apply time series strategy in pairs trading. This new model combines the advantages of time series models and non-directional trading strategy. In traditional pairs trading model, people use constant threshold to trigger trade

[25] Including the top 5 pairs.

and this value is subjective and constant over time. The major problem of this naïve model is that this subjective threshold cannot catch the dynamics of the spread between the pairs and therefore the trading performance is not optimized. In my model, the dynamics of the spread is caught by using non-constant thresholds which are calculated based on the most current information. Based on previous information the efficient and relatively accurate GARCH (1,1) model provides forecast of variation for the next trading period (next day in this paper) and this predicted variation is used to build dynamic thresholds. From the results we can see that this time series based strategy beats the naive constant threshold model and generates noticeable returns.

I used GARCH(1,1) model in this paper and the result is promising. The next step may be an extension from GARCH (1,1) to GARCH (p,q). In my future research I am going to try other more advanced time series models. A further examination of whether more complicated time series models improve the performance is an important question for future research.

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Table 5

Summary Statistics for the Top 20 Pairs

Spread	Mean	Std. Dev.	Skewness	Kurtosis	Dickey Fuller
FRT_SPG	-1.63e-09	.0694932	.2984633	3.551998	-5.282*
REG_SPG	-7.70e-10	.0805957	-.2491325	2.646239	-5.371*
MAC_REG	1.57e-09	.1015472	.223896	2.6248	-4.774*
NPG_NPM	1.41e-09	.1809379	-.0799789	2.770183	-5.626*
CMCSA_CMCSK	-2.98e-10	.1679888	1.58133	4.410258	-5.031*
AMB_PLD	-2.48e-10	.1834878	.1044347	2.51975	-6.887*
BXP_SLG	8.92e-10	.1170613	.1884893	2.427899	-3.872*
TCO_FRT	1.28e-09	.1287253	.7682123	5.610813	-3.451*
HIO_MHY	2.11e-09	.1352534	-.4002371	3.938504	-3.926*
BTI_ITY	-4.46e-11	.1743398	.1236597	2.508867	-5.097*
ARE_AMB	-2.53e-10	.1280959	.1127409	3.083197	-5.311*
AVB_BXP	-1.03e-09	.1317921	-.194347	2.651474	-3.789*
IFN_IIF	-3.89e-10	.1355652	1.078405	7.970653	-3.807*
ADVNA_ADVNB	1.82e-09	.1516715	1.749457	6.506454	-8.314*
ESS_BXP	-2.16e-09	.1426923	-.4067968	2.779833	-4.049*
BRE_ESS	2.84e-10	.1454214	.183783	2.47698	-4.644*
VNO_ESS	1.28e-09	.1722305	.1626103	2.517043	-3.695*
OFC_PSA	-7.24e-11	.1509805	-.5686995	2.756176	-3.594*
EWW_MXF	-2.38e-09	.1640672	.0649009	2.778703	-3.525*
DDR_MAC	-5.55e-10	.0112027	-.1140754	2.813373	-4.073*

Sample: January 01, 2000 to April 29, 2007. * Significant at the 1% level.

Table 6

GARCH (1,1) Estimation

Spread	α_0	α_1	θ_1
FRT_SPG	.0002439*	.7605913*	.1896429*
	(.0000344)	(.0957653)	(.0537789)
REG_SPG	.0002295*	.843056*	.1327086*
	(.0000402)	(.0957066)	(.0496894)
MAC_REG	.0001852*	.8513523*	.1463991 *
	(.0000298)	(.0922808)	(.0437996)
NPG_NPM	.0217849*	.8392988*	.0975894*
	(.0030357)	(.0991612)	(.0549154)
CMCSA_CMCSK	.0025467*	.8672314*	.1082528*
	(.0000354)	(.0892345)	(.0218926)
AMB_PLD	.0045346*	.8467475*	.1209234 *
	(.0013414)	(.0335399)	(.0234564)
BXP_SLG	.0001831*	.8196053*	.1839429*
	(.0000321)	(.0884272)	(.0580544)
TCO_FRT	.0002046*	.8763524*	.1318344*
	(.0000328)	(.1162951)	(.0605668)
HIO_MHY	.0002807*	.4082794*	.5995136*
	(.0000443)	(.0374364)	(.0215214)
BTI_ITY	.0001686*	.6531321*	.3609589*
	(.0000383)	(.0727173)	(.0286135)
ARE_AMB	.0003452*	.8112249*	.1860371*
	(.0000442)	(.0842958)	(.0310367)
AVB_BXP	.000159*	.6968305*	.3011946 *
	(.0000261)	(.0929389)	(.0390237)
IFN_IIF	.0000468*	.5929086*	.4330929*
	(8.91e-06)	(.0563305)	(.0256911)
ADVNA_ADVNB	.0000941*	.6799341*	.3079337*
	(.0000199)	(.090101)	(.0702249)
ESS_BXP	.0005581 *	.9233612*	.0548445
	(.0000603)	(.1056652)	(.0428582)
BRE_ESS	.0007262*	.9593607*	.0262469
	(.0001042)	(.109084)	(.0600046)
VNO_ESS	.0004903*	.8774402*	.1034875*
	(.0000607)	(.1163521)	(.0462367)

OFC_PSA	.0002222* (.0000421)	.7285743* (.0930678)	.2756713* (.0428019)
EWW_MXF	.0001661* (.000037)	.8216065* (.1074709)	.195684* (.0562614)
DDR_MAC	.0001768* (.00003)	.689994* (.0830771)	.3050286 * (.0307522)

Notes: * Significant at the 5% level. Numbers in parenthesis are standard errors. α_0 is the constant, α_1 is the coefficient for first order ARCH effect (autoregressive lags) for the spread series, and θ_1 is the first order GARCH effect (moving average lags) for the spread series.

Table 7

Excess Returns with Different Threshold Functions

Pairs	Top 5	Top 20
A. Excess return (with the threshold of 0.75 std)		
Excess return distribution		
Mean	.18945	.06234
Median	.04121	.03955
B. Excess return (with the threshold of 2 std)		
Excess return distribution		
Mean	.16235	.04903
Median	.03621	.03016
C. Relative performances		
Paired t-test	.02769*	.02566*
	(.01786)	(.01701)

Summary statistics of the annually excess returns on pairs between April 30, 2007 and April 30, 2008 (254 observations). I trade according to the rule that opens a position in a pair at the end of the day that normalized prices of the stocks in the pair diverge by 0.75 of the historical standard deviation (Panel A). The results in Panel B correspond to returns based on a threshold of 2 times of the historical standard deviation. All pairs are ranked according to least distance in historical price space. The ‘‘top n’’ portfolios include the n pairs with least distance measures. Top 20 pairs includes the top 5 pairs. Transaction costs are included. * Significant at the 10% level. Numbers in parenthesis are standard errors.

Table 8

Excess Returns of Pairs Trading Strategies

Pairs	Top 5	Top 20
A. Excess return distribution (Dynamic threshold method)		
Excess return distribution		
Mean	.21252	.07816
Median	.12251	.07655
Standard deviation	.25345	.17989
Skewness	1.54121	2.21231
Kurtosis	3.22342	9.84562
Minimum	.06032	-.08097
Maximum	.68752	.68752
B. Excess return distribution (Constant threshold method, 0.75 std)		
Excess return distribution		
Mean	.18945	.06234
Median	.04121	.03955
Standard deviation	.27678	.17352
Skewness	1.42150	3.11006
Kurtosis	2.92342	11.23522
Minimum	.01985	-.09345
Maximum	.58852	.58852

Summary statistics of the annually excess returns on pairs between April 30, 2007 and April 30, 2008 (254 observations). I trade according to the rule that opens a position in a pair at the end of the day that normalized prices of the stocks in the pair diverge 2 times of the predicted standard deviation (Panel A). The results in Panel B correspond to a strategy that constant threshold used across the whole trading period. All pairs are ranked according to least distance in historical price space. The ''top n'' portfolios include the n pairs with least distance measures. Top 20 pairs includes the top 5 pairs. Transaction costs are included.

Table 9

Excess Returns of Pairs Trading Strategies

Pairs	Retuns
A. Excess returns (Dynamic threshold method)	
FRT_SPG	.68752
REG_SPG	.57634
MAC_REG	.34235
NPG_NPM	.06032
CMCSA_CMCSK	.11241
AMB_PLD	.10126
BXP_SLG	.06356
TCO_FRT	.05678
HIO_MHY	.05216
BTI_ITY	.02865
ARE_AMB	.02788
AVB_BXP	.03979
IFN_IIF	.01098
ADVNA_ADVNB	.00986
ESS_BXP	-.06329
BRE_ESS	.01326
VNO_ESS	-.08097
OFC_PSA	.02012
EWW_MXF	.00186
DDR_MAC	-.02123

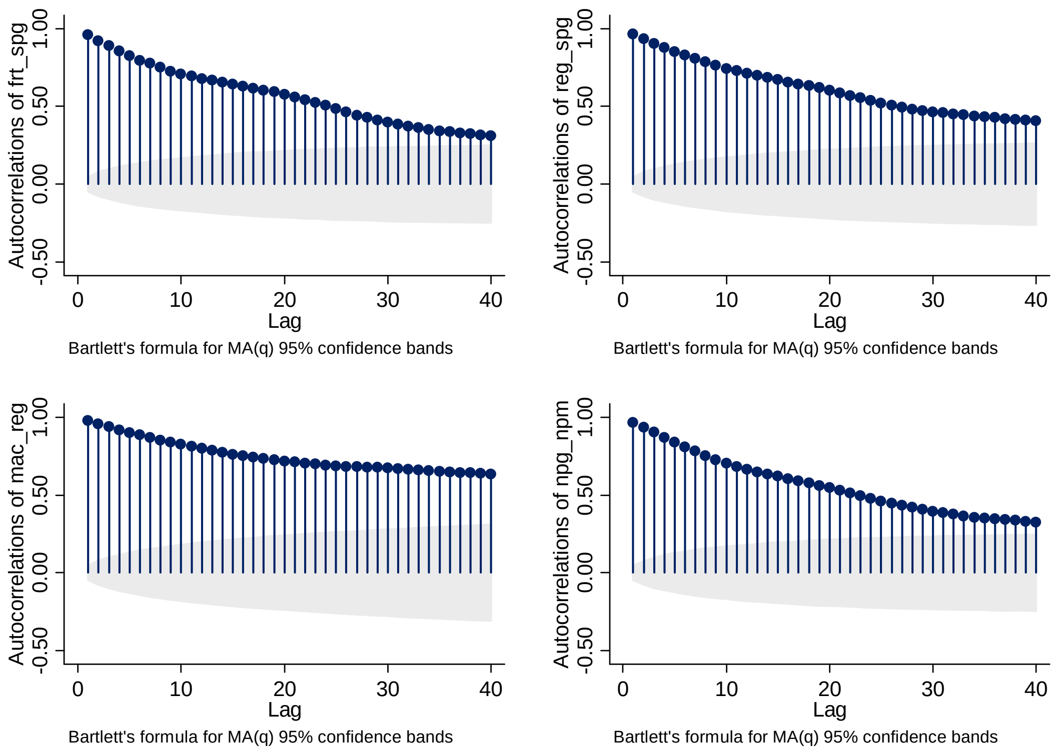
Pairs	Retuns
B. Excess returns (Constant threshold method, 0.75 std)	
FRT_SPG	.58852
REG_SPG	.45022
MAC_REG	.40235
NPG_NPM	.04987
CMCSA_CMCSK	.08976
AMB_PLD	.01985
BXP_SLG	.04321
TCO_FRT	.06235
HIO_MHY	.03087
BTI_ITY	.05080
ARE_AMB	.01987
AVB_BXP	.10211
IFN_IIF	.01657
ADVNA_ADVNB	.01021
ESS_BXP	-.09345
BRE_ESS	.00976
VNO_ESS	-.01098
OFC_PSA	.01987
EWW_MXF	-.00186
DDR_MAC	-.03865

C. Relative performances		
Dynamic threshold method beats constant threshold method	4/5	13/20
Paired t-test	.03032*	.02056*
	(.02001)	(.01686)

Summary statistics of the annually excess returns on pairs between April 30, 2007 and April 30, 2008 (254 observations). I trade according to the rule that opens a position in a pair at the end of the day that normalized prices of the stocks in the pair diverge 2 times of the predicted standard deviation (Panel A). The results in Panel B correspond to a strategy that constant threshold used across the whole trading period. All pairs are ranked according to least distance in historical price space. The ''top n'' portfolios include the n pairs with least distance measures. Top 20 pairs includes the top 5 pairs. Transaction costs are included. * Significant at the 10% level. Numbers in parenthesis are standard errors.

Figure 8

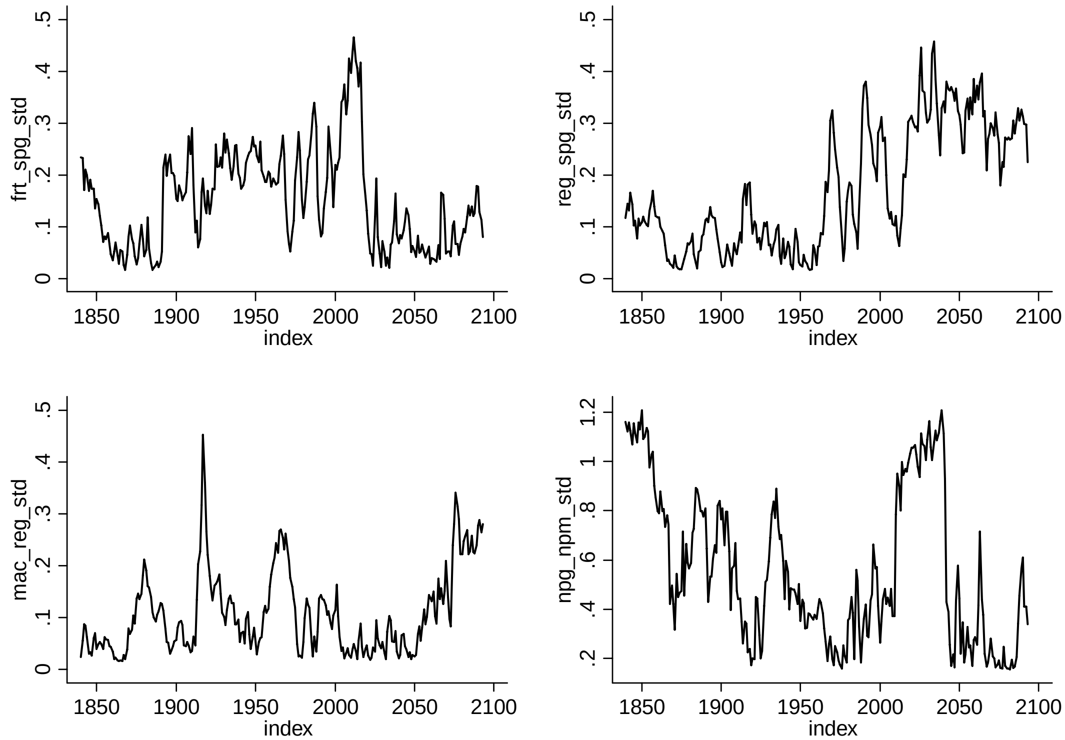
Autocorrelation Function Plots of the Spreads for the Top 4 Pairs



Sample: January 01, 2000 to April 29, 2007.

Figure 9

Predicted Conditional Standard Deviation for the Top 4 Pairs



Sample: April 30, 2007 to April 30, 2008.
Estimation is conducted using the sample from January 01, 2000 to April 29, 2007, or the first 1839 observations. The conditional standard deviation is predicted for the period April 30, 2007 to April 30, 2008 (observation 1840 to observation 2093).

Figure 10

Negative Cash Flow

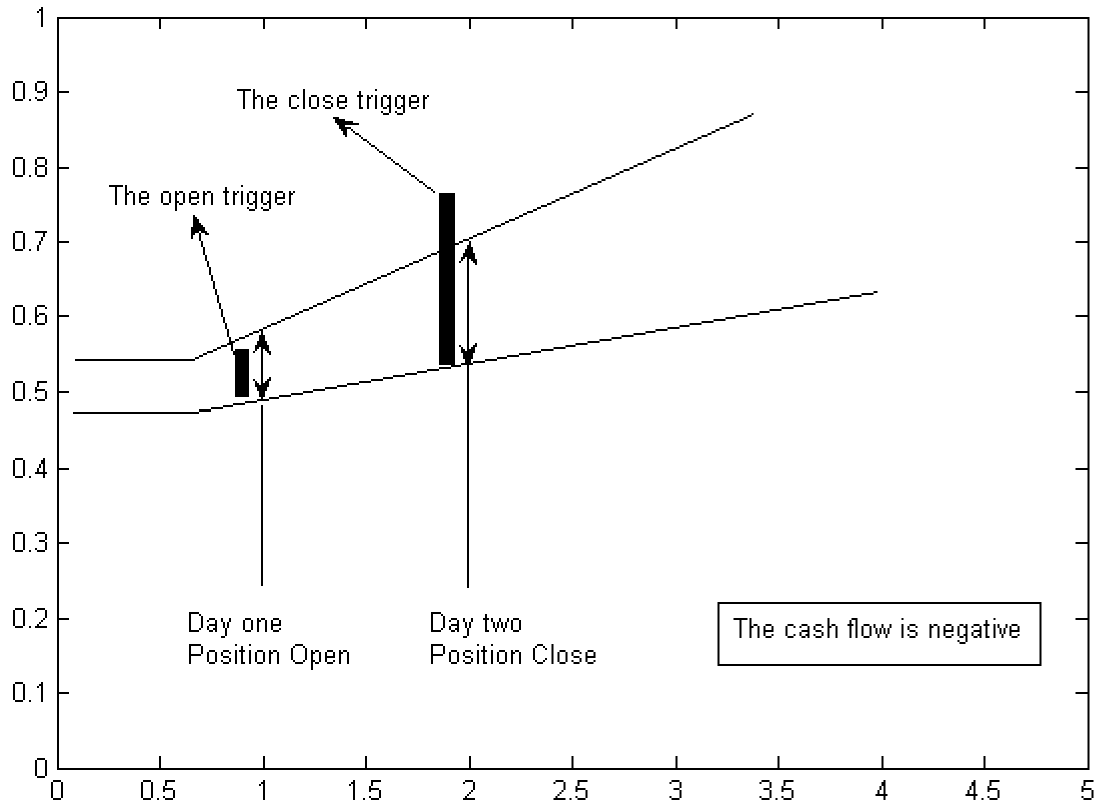


Figure 11

Constant Threshold Method

Sample: July 30, 2007 to October 30, 2007

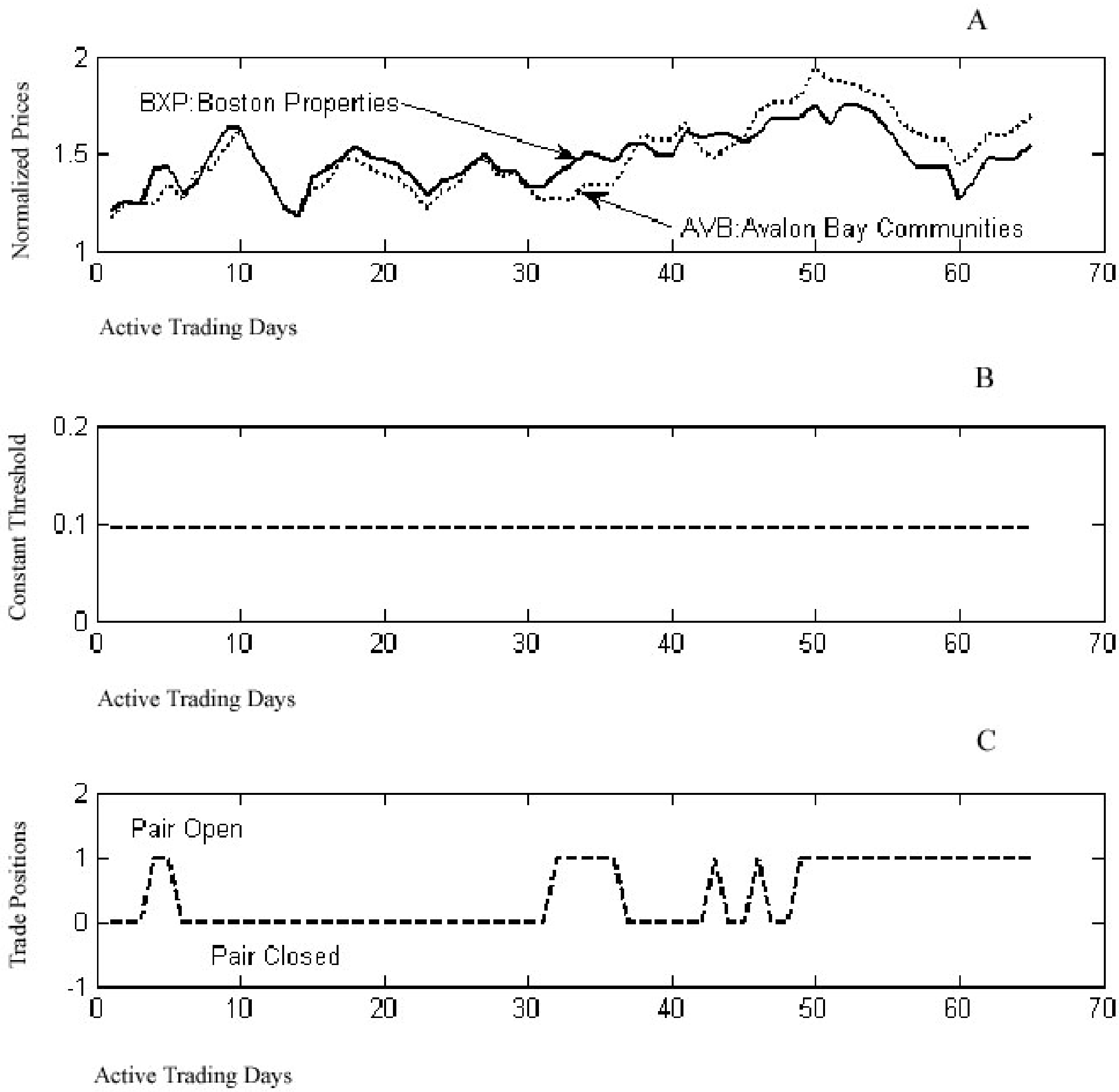
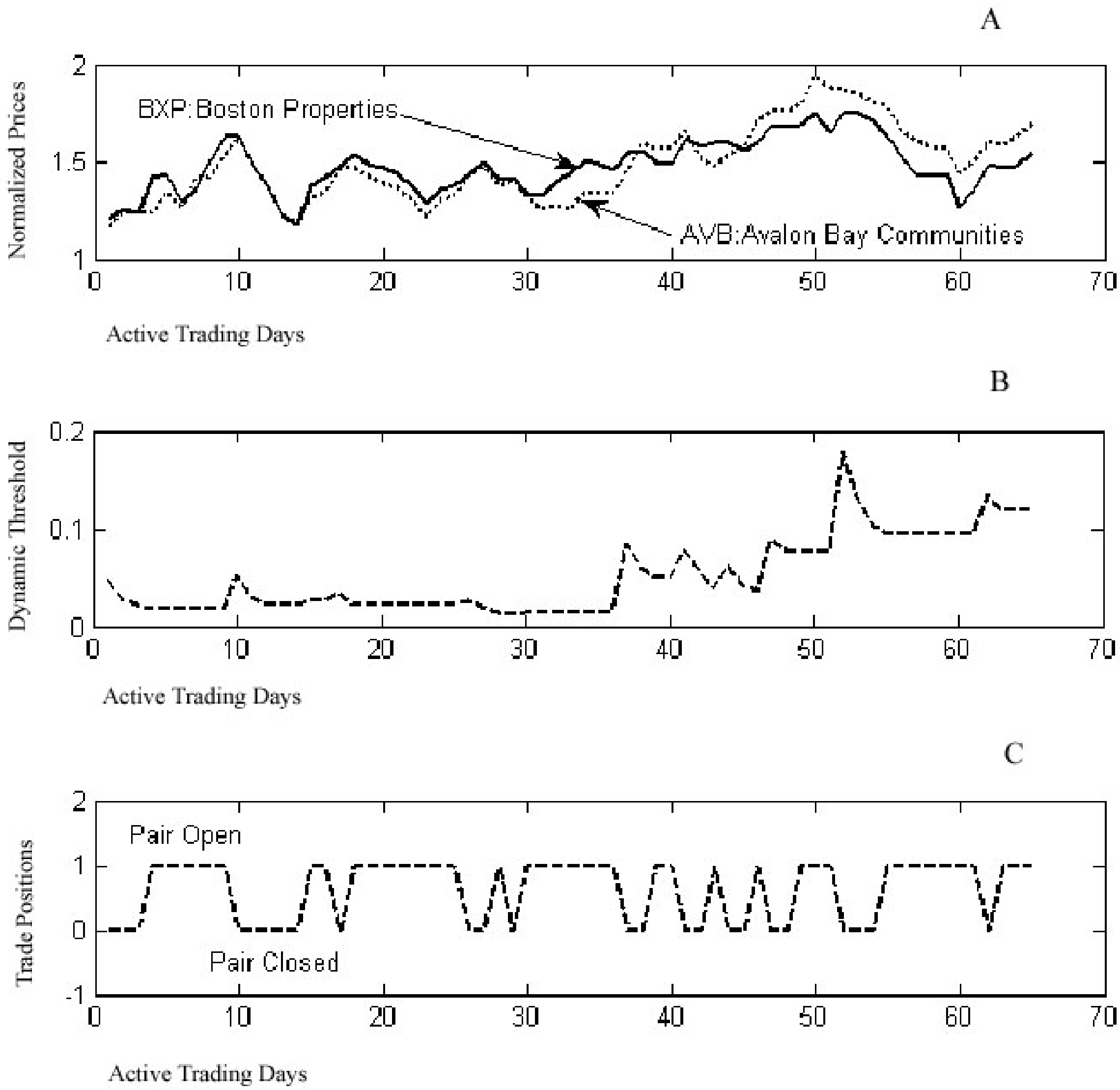


Figure 12

Dynamic Threshold Method

Sample: July 30, 2007 to October 30, 2007



APPENDIX

Tickers and company names

Tickers	Company names
ADVNA	Advanta Corp CLA
ADVNB	Advanta Corp
AMB	AMB Property Corp
ARE	Alexandria Real Estate Equities Inc.
AVB	Avalonbay Communities Inc.
BRE	BRE Properties Inc.
BTI	British American Tobacco plc
BXP	Boston Properties Inc.
CMCSA	Comcast Corp.
CMCSK	COMCAST CL A SPCL
DDR	Developers Diversified Realty Corp
ESS	Essex Property Trust Inc.
EWW	iShares MSCI Mexico Index
FRT	Federal Realty Investment Trust
HIO	Western Asset High Income Opportunity Fund Inc.
IFN	India Fund, Inc.
IIF	Morgan Stanley India Investment Fund, Inc.
ITY	Imperial Tobacco Group plc
MAC	Macerich Co.
MHY	Western Asset Managed High Income Fund Inc.
MXF	The Mexico Fund, Inc.
NPG	Nuveen Georgia Premium Income Municipal Fund
NPM	Nuveen Premium Income Municipal Fund 2 Inc.
OFC	Corporate Office Properties Trust Inc.
PLD	ProLogis
PSA	Public Storage
REG	Regency Centers Corporation
SLG	SL Green Realty Corp
SPG	Simon Property Group Inc.
TCO	Taubman Centers Inc.
VNO	Vornado Realty Trust