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A COMPUTERIZED MODEL OF LARGE-SCALE VISUAL INTERSTELLAR EXTINCTION

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ABSTRACT

A numerical algorithm has been developed for calculating three-dimensional visual interstellar extinction and its error from inputs of Galactic longitude, Galactic latitude, and distance. The code synthesizes the results of several published studies and can be used both for making corrections to individual observations and for correcting statistical samples. The studies have been modified to introduce a statistical extinction correction at large (poorly-sampled) distances, and to truncate values at the (assumed) edge of the Milky Way. When combined, the studies allow individual absorbing clouds in the interstellar medium to be identified within 5 kpc. Several observational selection biases are noted and discussed in view of this combined analysis. The FORTRAN-coded algorithm EXTINCT.FOR (v. 2.0.2) can be obtained via anonymous ftp from site ftp.mankato.msus.edu/pub/astro. © *1997 American Astronomical Society.* [S0004-6256(97)03611-X]

1. INTRODUCTION

The Galactic distribution of obscuring interstellar gas and dust distorts our view of objects both within the Galaxy and outside of it. This interstellar material is known to be clumpy, and concentrated in diffuse clouds that are for the most part distributed close to the Galactic plane.

In one of the great conundrums of astronomy, visual extinction identification is based primarily on deviant star and galaxy magnitudes and colors, while statistical properties of stars and galaxies can only be identified when corrections due to interstellar extinction have been made. Thus identification of light-absorbing material has been an iterative process coupled with the identification of statistical stellar and galaxian properties.

The distribution of Galactic visual extinction A_v was found by Parenago (1940) to obey a barometric law:

$$A_v = \frac{a_v \beta}{|\sin b|} \left(1 - e^{-(d|\sin b|)/\beta}\right), \tag{1}$$

where b is the Galactic latitude of the object being obscured, d is the object's distance, a_v is the differential extinction (mag/kpc) in the Galactic plane ($b=0°$), and β is the extinction scale height (pc). The Parenago relationship adequately describes large-scale extinction distribution perpendicular to the Galactic plane, but was designed to address neither Galactic longitude (l) extinction variations nor small-scale variations arising from dense interstellar clouds. The differential extinction a_v has been found to have an average value between 1.0 and 2.0 mag/kpc.

Many subsequent studies have addressed these oversights of the Parenago relationship in attempts to improve upon our understanding of the interstellar extinction distribution. The standard approach has been to parcel out the sky in angular cells, each defined by boundaries in Galactic coordinates (l,b). From the stars (or galaxies) in each cell, the visual extinction $A_v(l,b)$ can then be obtained as a function of distance $A_v(l,b,d)$. The angular size of the cells has varied from study to study, although in general each cell is generally chosen to be large enough to contain a statistically significant number of calibration stars at different distances (or galaxies, at distances beyond the Milky Way), yet small enough so that individual interstellar clouds can be resolved. The presentation of these different analyses in the literature has limited their direct comparison, as some have been presented graphically, while others have been presented as functions or as contour plots. Few have been reproduced as computer code.

We have attempted to combine results from a number of studies despite their differing formats and styles of presentation. The end-product is a FORTRAN subroutine that calculates the visual interstellar extinction A_v and its error σ_A given the inputs $l(°)$, $b(°)$, and d(kpc). It is our hope that, by combining the results of authors who have painstakingly made interstellar extinction measurements, a consistent picture of the local visual interstellar extinction can be obtained. It is further hoped that the resulting computer code can be used by others to correct visual magnitudes of individual stars or galaxies, so that the next step of iteration can be made in determining statistical stellar (galaxian) properties, as well as the structure of the Milky Way Galaxy.

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2. DATA AND CODING PROCEDURE

We have chosen for inclusion in this analysis a number of representative visual interstellar extinction studies. These are not meant to constitute a complete list of available studies: it is not our intention to present an exhaustive summary of analyses discussed in the literature. Instead, we desire primarily to identify a consistent view of the nearby visual interstellar extinction despite differences in the methodologies and presentation styles of these representative papers. The studies chosen generally span large angular and distance scales, and tend to overlap one another to a considerable degree. Each study has been converted to an independent FORTRAN subroutine so that the results could be combined. The program therefore has room to evolve: we will continue to search for other papers for inclusion in future updates of this analysis.

The studies we have chosen for inclusion at this time are those of FitzGerald (1968), Neckel & Klare (1980), Berdnikov & Pavlovskaya (1991), Arenou *et al.* (1992), and a high-galactic latitude study composed of the combined works of many authors. Two of these studies (FitzGerald 1968; Arenou *et al.* 1992) are essentially all-sky studies, two (Berdnikov & Pavlovskaya 1991, and the combined high-latitude study) are high Galactic latitude studies, and one (Neckel & Klare 1980) is a low Galactic latitude study. The studies have varying degrees of angular resolution and distance limits, use formats ranging from purely analytical to strictly numerical, and produce results that are not obviously correlated with one another.

All studies have been modified to provide data beyond their formal distance limits, which allows for use of the code in statistical stellar studies. This has been done by assuming a statistical differential extinction value of $a_v = 1.5 \pm 0.5$ magnitudes per kpc for distances at which the studies do not provide data. This value is adjustable by the user.

The reader should keep in mind that, despite the different formats used and results obtained from these studies, very similar data sets have been used (stars and galaxies along with their corresponding colors, magnitudes, and spectral/morphological classification types) so that the results are not statistically independent. We discuss each of these studies and how it has been implemented as computer code below.

2.1 The Study of FitzGerald (1968)

We choose the often-cited study of FitzGerald (1968) for conversion to computer code because of its good angular resolution (angular cells in the Galactic plane are typically 12° in diameter) and because its description of extinction both in and perpendicular to the Galactic plane allows it to be used (with modification) over the full sky. Angular cells vary in size and shape in this and subsequently-discussed studies; we define angular resolution as the diameter of a sampled sky area containing the solid angle of an average cell.

FitzGerald maps interstellar extinction using color excesses (CE) of 7835 O to M stars found in the first version of the Photoelectric Catalogue of Blanco and FitzGerald (McCuskey 1966), as well as color excesses of 208 open clusters. The color excesses are mapped within 10° of the Galactic plane for 74 Galactic longitude zones (Table II and Fig. 3 of FitzGerald) to distances of 2–3 kpc, as well perpendicular to the Galactic plane within roughly 1 kpc of the Sun.

In converting these color excesses to computer code, the color excess vs. apparent magnitude diagrams in the FitzGerald Fig. 3 cells have been enlarged so that color excess in the Galactic plane can be obtained as a function of distance. An increase in the color excess is produced by absorbing material in the line of sight, while the color excess is constant when no absorbing material is present (since this is an integral relationship, it can never decrease with distance). Each cell is represented by a series of distance regions in which a linear slope CE/d has been assumed. When data were not available (e.g., at distances where the color excesses were not known), an average color excess of 0.5 mag per kpc was assumed, along with a 0.17 mag per kpc error.

The extinction A_v is obtained from the color excess CE by the relationship $A_v = R$ (CE). In making this conversion we assume (as FitzGerald does) a value of 3.0 for the ratio of total to selective absorption R.

The distribution perpendicular to the Galactic plane for planar distances within roughly 1 kpc is modeled by an exponentially decreasing density with half-widths typically between 40 and 100 pc (Table I and Fig. 2 of FitzGerald) for eleven Galactic latitude zones. We have assumed a half width of 65 pc for FitzGerald's zone g, since no half width was given for this zone other than what can be estimated from FitzGerald's Figure 2. These half widths are assumed to extend from the Sun to the nearest distance zone at or just beyond the maximum distance given in FitzGerald's Table I. Beyond these distances, we assume half widths of 114 pc (Parenago 1945; Sharov 1963), since this value is mentioned as a reasonable average by FitzGerald.

Extinction out of the Galactic plane is obtained (using the Parenago law described by FitzGerald) by combining the radial behavior in the plane with the functional behavior perpendicular to the plane. This produces a model in which extinction from clouds in the Galactic plane is assumed to exponentially decrease as a function of distance from the plane. Extinction out of the plane has the form

$$A_v = \sum_i \frac{a_{vi}\beta}{|\sin(b)|} \left(e^{-d_{0i}|\tan(b)|/\beta} - e^{-d_{0i+1}|\tan(b)|/\beta}\right), \qquad (2)$$

where the differential extinction out of the plane is summed over i clouds in the Galactic plane. Here, d_{oi} is the distance in the Galactic plane to the front of the cloud, d_{oi+1} is the distance in the Galactic plane to the back of the cloud (or to a specified distance inside the cloud), and a_{vi} is the differential extinction that the cloud would have if it were in the Galactic plane.

It should be noted that in using the Parenago law variation described by Eq. (2), FitzGerald assumed that his stellar sample belonged to a single Galactic disk component, and that metallicity differences did not result in intrinsic stellar color variations. Recent studies demonstrate the existence of an additional Galactic thick disk component (e.g., Friel 1987; von Hippel & Bothun 1993; Ojha *et al.* 1996), which indi-

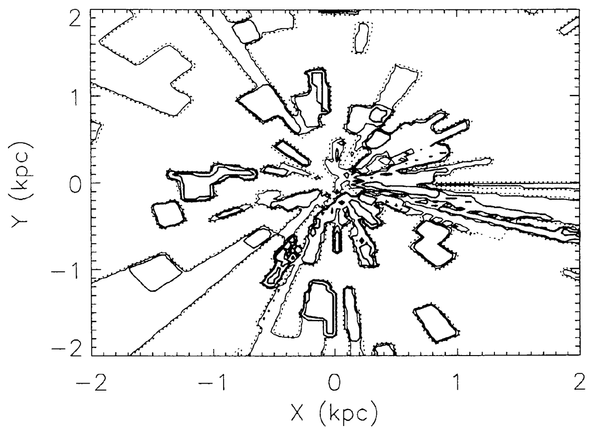

FIG. 1. Differential visual extinction a_v (mag/kpc) in the Galactic plane as obtained from coding the study of FitzGerald (1968). In this and all subsequent Galactic plane plots, the Sun's location is at the origin, the Galactic center is in the $-y$ direction, and the direction of Galactic rotation is in the $+x$ direction. Plot contours indicate differential extinction levels of 0.5 (dotted line), 1, 2, and 5 mag/kpc (solid lines of increasing thickness). Each Galactic plane study has been modified to have a statistical extinction of $a_v = 1.5$ mag/kpc at distances exceeding the study's quoted distance maximum. Figure 1 is in excellent agreement with FitzGerald's Fig. 4, indicating that analysis has been accurately represented by this computer code.

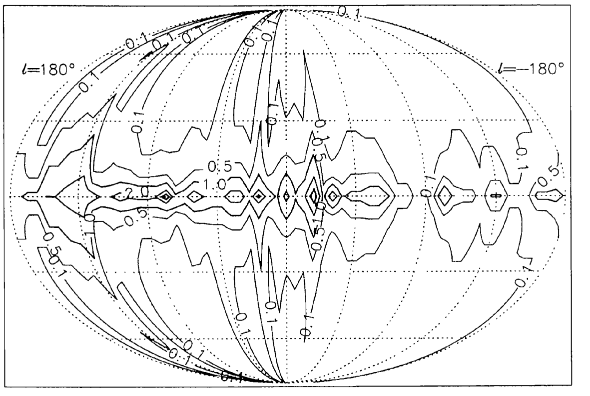

FIG. 2. Total visual extinction A_v (magnitudes) in Galactic coordinates to a distance of 1 kpc as obtained from the FitzGerald (1968) study. Plot contours are 0.1, 0.5, 1.0, 2.0, and 3.0 mag.

cates that the accuracy of the FitzGerald model decreases at higher Galactic latitudes.

Extinction errors have been obtained from FitzGerald's 74 Galactic longitude zones. The average color excess error σ_{CE} has been estimated as a function of color excess CE using a coarse binning, and is well approximated (with a correlation coefficient of 0.98) by $\sigma_{CE} = 0.059(CE)^2 + 0.218(CE) + 0.044$. This error is multiplied by a total to selective absorption ratio of $R = 3.0$ to produce the extinction error.

Figure 1 indicates the differential visual extinction (mag/kpc) in the Galactic (x,y) plane to a distance of 2 kpc, as obtained from our first subroutine. We use the standard convention that the Sun is at coordinates (0,0), the Galactic Center is aligned with the $-y$ axis, and the $+x$ axis lies in direction of Galactic rotation for the Local Standard of Rest. Interstellar clouds have positive values of varying degrees. This figure is in excellent agreement with FitzGerald's Fig. 4, indicating that his analysis has been accurately represented by our computer code. Figure 2 demonstrates the appearance of this extinction on the celestial sphere (l,b) sampled to a distance of 1 kpc.

2.2 *The Study of Neckel & Klare (1980)*

The 1980 study of Neckel and Klare utilizes 11 072 objects (of which 7565 are O and B stars) with MK spectral types, *UBV* observations, and β values to map visual extinction for 325 cells within 7.6° of the Galactic plane to distances of typically 3–4 kpc. The study's unique strength is the high angular resolution it provides (typical cell diameter is 2°), as the angular cells into which the sky is divided were chosen to have complex shapes as a result of (a) maintaining a large enough stellar sample to ensure reasonable counting statistics, and (b) defining cloud boundaries as accurately as possible. This approach also provides the greatest challenge in converting the study's results into computer code.

We have used a time-consuming procedure to convert the results of this study into computer code. The general procedure is described below.

First, the extinction vs. distance diagrams provided by Neckel & Klare as their Fig. 6 have been scanned as computer images. Although a functional form for the extinction vs. distance has been provided by the authors in many diagrams, there are others in which no estimates or only partial estimates are available. Thus we have had to estimate the extinction versus distance function in many cases. In doing so, we have assumed linear extinction in clouds, and have simply estimated cloud boundaries to the best of our ability. We have used the extinction provided by the authors in the extinction vs. distance diagrams (or our own estimates, when none were provided by the authors) along with the identifiable data points (scanning resolution has not allowed us to uniquely identify all individual data points) to estimate σ_A as a function of A_v.

Second, the angular cells identified by Neckel & Klare as their Fig. 5 have been scanned as computer images. The boundaries of these regions are very complex, and are difficult to describe in the form of simple computer code. Thus we have subdivided each complex cell into an appropriate number of rectangular and triangular subcells. Each subcell is linked to the appropriate extinction vs. distance diagram described by Neckel & Klare.

A number of blank cells exist in the paper for which no extinction vs. distance information is available. One of two conditions is met in each of these cells: either the extinction is so great that insufficient stellar counts prevent a meaningful extinction versus distance function to be produced, or the extinction is so small that the region was thought by Neckel & Klare to be uninteresting. We choose to ignore these blank cells rather than to account for them, since there are no data for the high-extinction zones. We do note, however, that the vacant cells lead to an underestimate of the overall extinction, since the highest-extinction regions are not available.

Figure 3 indicates the differential visual extinction (mag/

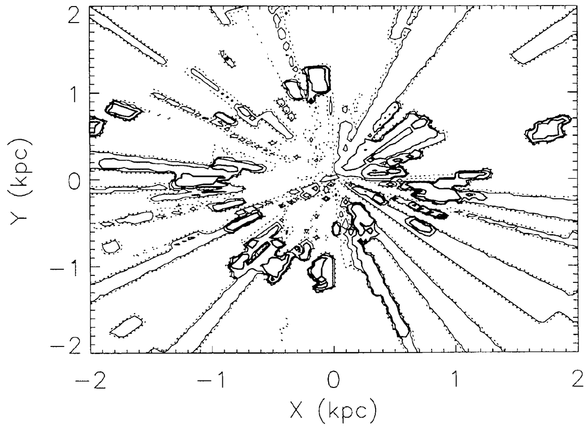

FIG. 3. Differential visual extinction a_v (mag/kpc) in the Galactic plane as obtained from coding the study of Neckel & Klare (1980). Plot contours indicate differential extinction levels of 0.5 mag/kpc (dotted line), 1, 2, and 5 mag/kpc (solid lines of increasing thickness).

kpc) in the Galactic (x,y) plane to a distance of 2 kpc, as obtained from our second subroutine. This figure is in excellent agreement with Neckel & Klare's Fig. 9. Figure 4 demonstrates the appearance of this extinction on the celestial sphere (l,b) sampled to a distance of 1 kpc. The high angular resolution of the Neckel and Klare study (relative to the FitzGerald study) is visible in these figures.

2.3 *The study of Berdnikov & Pavlovskaya (1991)*

Berdnikov & Pavlovskaya (1991) use photoelectric magnitudes of over 1000 high-luminosity stars in small Kapteyn areas to develop an analytical form for high Galactic latitude visual extinction ($|b|>10°$) between Galactic longitudes 65° and 165°. We have chosen this study primarily because of the analytical form in which the results are presented, but cannot estimate its angular resolution since results obtained from small selected areas have been averaged over and applied to much larger angular areas.

The extinction function is given by Berdnikov & Pavlovskaya as

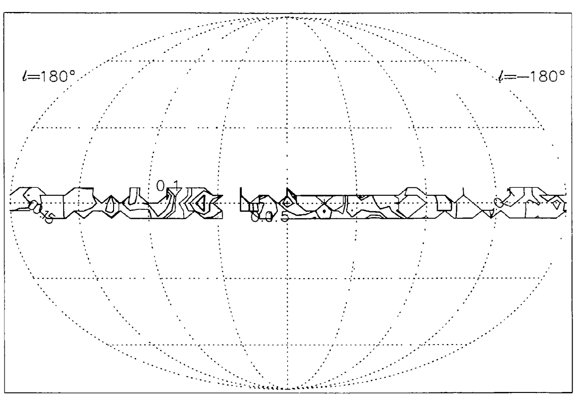

FIG. 4. Total visual extinction A_v (mag) in Galactic coordinates to a distance of 1 kpc as obtained from the Neckel & Klare (1980) study. Plot contours are 0.1, 0.5, 1.0, 2.0, and 3.0 mag.

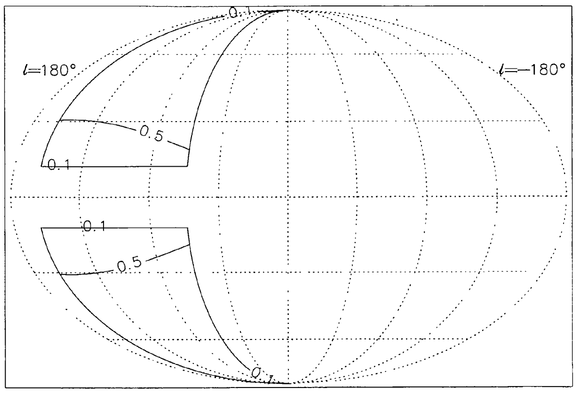

FIG. 5. Total visual extinction A_v (mag) in Galactic coordinates to a distance of 1 kpc as obtained from the Berdnikov & Pavlovskaya (1991) study. Plot contours are 0.1, 0.5, 1.0, 2.0, and 3.0 mag.

$$A_v=\frac{[\alpha\ \sin(l-\phi)+a_0]\beta d}{|\sin b|}\,(1-e^{-(d|\sin b|)/\beta}), \qquad (3)$$

with $a_0=1.09\pm0.41$ (differential extinction in the Galactic plane), $\beta=0.17\pm0.02$ (the extinction scale height), $\phi=64°\pm31°$, and $\alpha=0.49\pm0.36$. The latter two constants account for the extinction's Galactic longitude dependence.

The function is valid to distances of 1.5 kpc, with an estimated error of 0.41 mag. Beyond 1.5 kpc in this range of 1 and b, we have assumed that the extinction obeys Eq. (1), with a_0 and β as given for use in Eq. (4). The total extinction for $d>1.5$ kpc is obtained by adding Eq. (4) (evaluated at $d=1.5$ kpc) to Eq. (3) with $i=1$, $d_{01}=1.5$ kpc, $d_{02}=d$, and $a_{v1}=a_0$. The extinction error for $d>1.5$ kpc is assumed to be (1/3) $A_v(d>1.5\ \text{kpc})$, and the total extinction error is simply this error added in quadrature to 0.41.

Figure 5 demonstrates the appearance of extinction from this study on the celestial sphere (l,b), sampled to a distance of 1 kpc. The study produces differential and integral extinction values that smoothly change with l, b, and d, such that the Berdnikov & Pavlovskaya study appears to have a better angular and distance resolution than the other studies. However, this is only a result of the choice of the form of the extinction function.

2.4 *The Study of Arenou et al.* (1992)

The 1992 study of Arenou, Grenon, & Gómez has been published in a format easily converted to computer code, and has in fact been released with e-mail access to the program. Thus, inclusion of this study as a FORTRAN subroutine has been easy, and has only required statistical modifications at large distances (as noted previously).

Arenou *et al.* (1992) use 58 000 stars in the combined INCA (Grenon 1989) and SIMBAD (Egret 1985) databases that have B magnitudes, V magnitudes, and MK spectral types to determine visual extinction values. Their study subdivides the sky into 199 Galactic longitude and latitude cells, so that a typical cell has an angular diameter of 46° (but varying with galactic latitude). The extinction in each cell is fit by a quadratic function of distance out to an absorbing layer distance limit, and by a linear function beyond that.

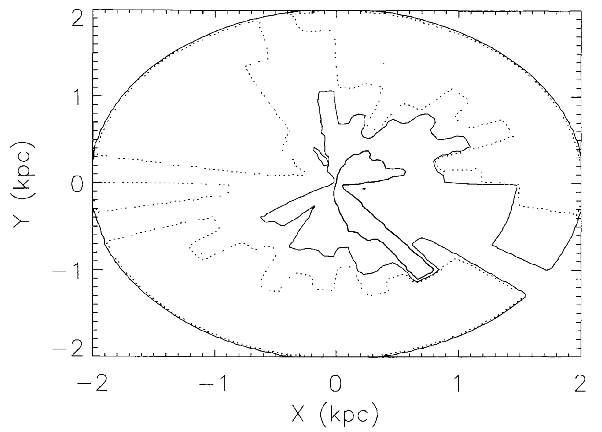

FIG. 6. Differential visual extinction a_v (mag/kpc) in the Galactic plane as obtained from coding the study of Arenou *et al.* (1992). Plot contours indicate differential extinction levels of 0.5 (dotted line), 1, 2, and 5 mag/kpc (solid lines of increasing thickness).

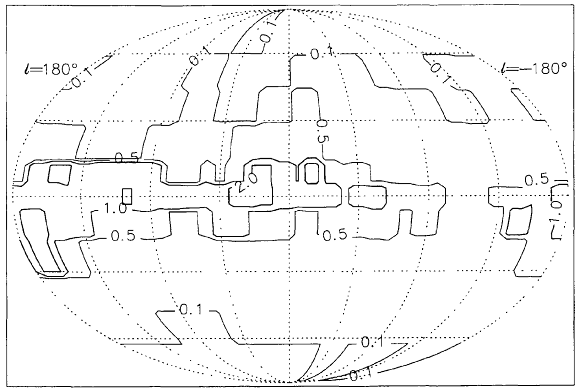

FIG. 7. Total visual extinction A_v (magnitudes) in Galactic coordinates to a distance of 1 kpc as obtained from the Arenou *et al.* (1992) study. Plot contours are 0.1, 0.5, 1.0, 2.0, and 3.0 mag.

An error analysis algorithm is also provided in the paper for each cell. At a specified distance d, an extinction error σ_{A1} can be calculated, with a maximum value occurring at $d=1.5$ kpc. Since we approximate the error beyond this distance as $\sigma_{A2}\approx 0.5$ mag/kpc$\times[d(\mathrm{kpc})-1.5\ (\mathrm{kpc})]$, the extinction error σ_A for $d\geqslant 1.5$ kpc is obtained simply by adding σ_{A1} and σ_{A2} in quadrature.

Figure 6 indicates the differential visual extinction (mags/kpc) in the Galactic (x,y) plane to a distance of 2 kpc, as obtained from our third subroutine. Figure 7 demonstrates the appearance of this extinction on the celestial sphere (l,b) sampled to a distance of 1 kpc. The quadratic extinction model produces curved surfaces in the total extinction versus distance, as contrasted with flat-topped cloud regions produced by the FitzGerald and Neckel & Klare analyses.

It is apparent that the extinction normalization obtained from the Arenou *et al.* study is very different from that in the FitzGerald study. Furthermore, from Fig. 6 it can be seen that the differential extinction has peak values very near the Sun, and generally decreases with distance after this until the statistical value is introduced at 1.5 kpc. This general decrease in the extinction with distance (also present in the FitzGerald and Neckel & Klare studies) will be addressed in Sec. 3.

2.5 The High-Galactic Latitude Sample

High-Galactic latitude cloud properties are different than cloud properties in the Galactic plane. High latitude clouds are smaller, less massive, and form less complex structures than Galactic plane clouds (e.g., Blitz *et al.* 1984). These clouds are rare enough and small enough that few clouds should lie along a line of sight, so that each cloud can be treated individually in the extinction code. For this reason, we have attempted to compile information on a large number of high-Galactic latitude clouds described in the literature.

This compilation was originally designed around clouds found in the detailed work of Penprase (1992), who uses observations of 320 stars to obtain sizes of and extinction for 25 high-Galactic latitude clouds. This list has been expanded in version 2.0.2 of the extinction program to include additional clouds from the studies of Magnani *et al.* (1985), Keto & Myers (1986), Désert *et al.* (1988), and Odenwald (1988) in order to improve upon the completeness of the high-latitude sample. We also have attempted to include a number of mid-Galactic latitude, high-extinction cloud complexes; namely, those associated with Gould's belt (e.g., Stothers & Frogel 1974). Lupus cloud values have been taken from Hughes *et al.* (1993), Taurus cloud values from Kenyon *et al.* (1994), Perseus cloud values from Cernis (1990, 1993), Corona Australis and Ophiuchus/Scorpio complex values from Rossano (1978a, 1978b), and Orion complex values have been roughly estimated from Kutner *et al.* (1978).

Cloud contours are identified from a combination of H I observations, CO observations, *IRAS* 100 μm images, and/or star counts. Simple Galactic (l,b) boundaries are then extracted from these contours. This assumption tends to oversimplify the complexity of cloud structure; however, this also allows each cloud to be practically incorporated in the extinction subroutine. Angular cloud resolutions vary; they are typically larger than 1/2° and smaller than 5° (with the exception of some of the Gould's Belt clouds). In reproducing the properties of the high-latitude clouds, the angular regions are often elongated and wispy in nature.

Cloud extinctions and errors are obtained from the referenced sources. Extinction is assumed to be constant across a cloud's angular surface, so that angular resolution is limited to the typical extent of an individual cloud. Exceptions to this approach are provided for mid-latitude clouds, which are larger and have more complex structures than high-latitude clouds. Still, this assumption prevents the tool from providing high-resolution information about the distribution of extinction arising within individual clouds and cloud complexes. Errors are relatively large, indicating both the difficulty in making accurate measurements and the large variation in extinction over cloud angular extents. The smallest errors ($\sigma_{A_v}=0.24$) are primarily from the well-calibrated work of Penprase (1992), based on a mean error of 0.08 mag in E$(B-V)$ and assuming a value of 3.0 for the ratio of total to selective absorption.

Cloud distances and line-of-sight diameters have been also been taken from the aforementioned references. We note that many high latitude studies place very rough estimates on both quantities; several studies assume that all cloud distances are equally 100 pc (although the Penprase study indicates that the actual volume sampled is considerably larger, with many clouds at distances of 200 pc or greater), and line-of-sight diameters often appear to be underestimated.

We have chosen to estimate a zero magnitude extinction for some high latitude regions devoid of clouds. This is done on the basis of three observations: (a) in the line of sight, observers generally identify no measurable extinction between clouds, (b) clouds have fairly distinct boundaries (indicating that regions of much lower extinction surround them), and (c) some of the identified clouds have low extinction (implying that the background extinction is even lower). In general, the background extinction is typically less than 0.15 mag, based on both the lowest-extinction high-latitude clouds observed (with $A_v \leq 0.2$ mag) and on the general background extinction obtained at high-Galactic latitudes from H I surveys (typically 0.0–0.2 mag). Since the smallest formal error we have obtained along cloud lines of sight ($\sigma_{Av} = 0.24$ mag; this error is also valid for intercloud regions) is larger than other background extinction measures, we feel justified in assuming that $A_v = 0.0$ mag for some high latitude unsampled regions.

However, we cannot assume that the background extinction is zero at all latitudes. We assume that the high-latitude sample is relatively complete for nearby (within roughly 200 pc), opaque, high-Galactic latitude clouds. The clouds in this volume are few and cover a small angular extent. However, we do not know how many clouds might lie along a line of sight at lower latitudes, so we choose to consider our sample complete above one Galactic scale height (assumed to be 114 pc). Thus, we assume that the background extinction is zero for $|b| > 34.75°$; otherwise, we do not calculate background extinction in unsampled areas.

Figure 8 demonstrates the appearance of extinction from these high-Galactic latitude clouds on the celestial sphere (l,b), sampled to a distance of 1 kpc.

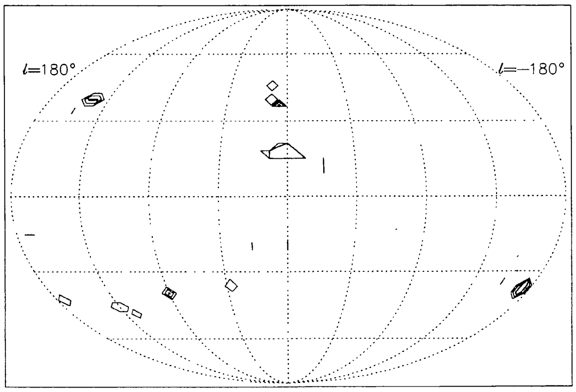

FIG. 8. Total visual extinction A_v (mag) in Galactic coordinates to a distance of 1 kpc as obtained from combining individual high- and mid-Galactic latitude clouds from the studies of Penprase (1992), Magnani *et al.* (1985), Keto & Myers (1986), Désert *et al.* (1988), Odenwald (1988), Hughes *et al.* (1993), Kenyon *et al.* (1994), Cernis (1990, 1993), Rossano (1978a, 1978b), and Kutner *et al.* (1977). The background extinction to this distance is assumed to be $A_v = 0.0$ for $|b| > 34.75°$, and is not calculated for areas devoid of clouds nearer to the Galactic plane. Contours in this plot are 0.1, 0.5, 1.0, 2.0, and 3.0 mag. It should be noted that many smaller clouds do not show up in this low-resolution plot.

3. THE SUBROUTINE

The FORTRAN subroutine EXTINCT.FOR (v. 2.0.2) produced from combining these five analyses requires 545 kb of storage (unzipped), and can be obtained via anonymous ftp from ftp.mankato.msus.edu/pub/astro. The subroutine (66 kb *g* zipped), a Microsoft Works® 3.0 copy of the submitted version of this manuscript, PostScript® copies of the submitted figures (*g*-zipped), and a text help file can be found at this address. It is intended that these will be periodically updated.

Given inputs of (l,b,d) at epoch 1950.0, the subroutine returns the overall visual extinction and its error obtained from available analyses, the extinction values and errors from all individual studies, and the maximum extinction value obtained along with its corresponding error. The overall visual extinction is an average obtained from the subroutines, with extinction from individual high latitude clouds chosen to supercede this when such values are provided (note: this v. 2.0 procedure replaces the average value obtained in v. 1.0).

The user can change a number of statistical settings in the main subroutine: the default statistical differential extinction (1.5 mags/kpc), the ratio of total to selective absorption R (used primarily in the FitzGerald subroutine and preset to 3.0, although R has clearly been assigned some value to produce extinctions found in the other subroutines), and the size of the confining Galactic disk (the model of which is described below).

It should be noted that formal extinction errors are often large. Combining analyses allows mean extinction values to be found, but the statistically-dependent datasets used in the analyses do not allow the formal error to be decreased (the error returned from the subroutine is the mean of the individual study errors). This simply indicates that formal measurement errors have in part kept observers from agreeing on detailed descriptions of interstellar extinction. This is in part exacerbated by the use of Gaussian rather than Poisson statistics.

The code can be used as an aid with which to model large-scale statistical Galactic or extragalactic distributions, or to obtain extinction for individual stars or galaxies. In the latter case, one needs only to find the distance d to the object from iterative solution of the distance modulus equation

$$V - M_v = 5 \log d - 5 + A_v(l,b,d). \quad (4)$$

It should be mentioned that these statistical corrections prevent the user from accurately reproducing extinction described in studies other than the ones used here. For example, the user will not be able to reproduce low extinction values known to exist in the direction of Baade's window at $l \approx 1°$ and $b \approx -3°$ (e.g., Stanek 1996), since the angular resolution of the studies used is not fine enough to identify this feature.

As mentioned previously, introduction of statistical extinction values at large distances from the Sun results in an increasingly poor approximation to the actual extinction distribution. In order for this code to be used in correcting magnitudes of extragalactic objects we have introduced an artificial boundary to the Galaxy by assuming a 15 kpc Galactic Disk radius and an 8.5 kpc Sun-Galactic Center distance. The maximum distance d_{max} (kpc) to which the absorbing material extends in the model is therefore

$$d_{max}=8.5\{\cos(l)\cos(b)+\sqrt{\cos^2(l)\cos^2(b)+2.114}\}. \quad (5)$$

4. ANALYSIS

The extinction model reproduces the most important large-scale properties of visual Galactic extinction: it is concentrated towards the Galactic plane, it varies as a function of Galactic longitude and Galactic latitude, and distinct opaque clouds and cloud complexes are present. However, some inconsistencies exist between results obtained from the various studies used in obtaining the model. Figures 1 through 8 demonstrate that extinction measurements can vary (often considerably) from study to study, even though the available stellar and galaxian data used to extract extinction measurements are for the most part common. Inconsistencies can be explained via calibration differences and the angular resolution and binning defined for each study.

Stellar absolute magnitude and intrinsic color calibrations can lead to fairly different distance estimates for a cloud, even if the same data set is used. An additional source of distance error arises from assuming an average R value, even though it can vary across interstellar medium environments. In some studies these systematic errors have been identified, while in others they have not.

The performance of the extinction code can be tested by comparing to extinction measurements obtained by other authors from visual, radio, and/or infrared observations. It should be noted that high-latitude cloud boundaries have been defined from data of these types, and correlative analyses described in the referenced papers have been discussed previously. We therefore examine general model characteristics both in the Galactic plane and at high-Galactic latitudes, since these represent well-studied extremes. We will use the results to comment on the model at middle Galactic latitudes, where studies are not as complete.

4.1 Extinction in the Galactic Plane

We can simultaneously obtain a map of visual extinction in the Galactic plane using pertinent subroutines. We have chosen to demonstrate the results obtained for differential visual extinction in the Galactic plane ($b=0°$) and within 2 kpc of the Sun in Fig. 9. Since the map is in the Galactic plane, the program makes use only of the FitzGerald (1968), Neckel & Klare (1980), and Arenou *et al.* (1992) analyses. There is at best a weak correlation of the interstellar medium obtained in this manner with three-dimensional Milky Way spiral structure found from either H I or young stellar objects

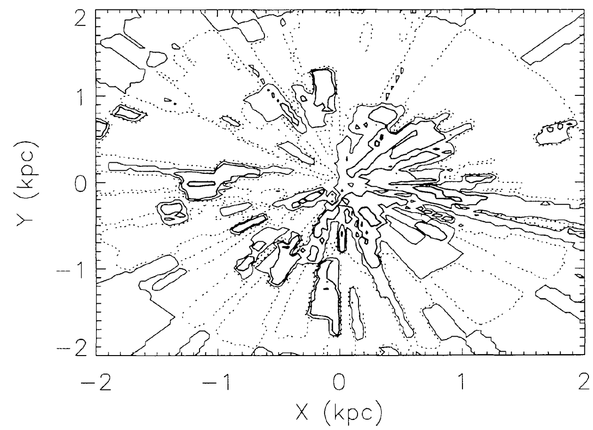

FIG. 9. Differential visual extinction a_v (mag/kpc) in the Galactic plane as obtained from combining the results of all planar studies with equal weights. Plot contours indicate differential extinction levels of 0.5 (dotted line), 1, 2, and 5 mag/kpc (solid lines of increasing thickness).

(e.g., Burton 1974; Vogt & Moffat 1975; Humphreys 1976). We have not performed a correlative study in making this statement; such an analysis is fairly complex and a visual comparison suggests that strong statistical correlations are not present.

The dominant feature observed in these figures is radial structure occurring due to inaccuracies in measuring stellar distances along a line of sight (e.g., Scheffler & Elsässer 1988). Since each study has used a different set of stellar absolute magnitude calibrations to obtain distances, this tends to distort features that may exist along a sight-line. However, a number of distinct cloud structures are clearly identified in the Galactic plane, such as opaque cloud complexes 0.5 kpc away towards $l\approx 30°$, 1.3 kpc away towards $l\approx 350°$, and 1 kpc away towards $l\approx 190°$.

The radial structure bias and complex cloud shapes make it difficult to clearly identify large-scale cloud features. However, it is possible that a long narrow band of absorbing material roughly 500 pc wide extends in the directions 1 $\approx 80°$ and $1\approx 260°$. If not due to selection biases, extinction from this band could be due to the Local Spiral Arm. Such an interpretation is debatable, as extinction due to the (external to the Sun) Perseus and (internal to the Sun) Sagittarius arms are not identifiable.

In cases where the binning has been chosen based on an a priori identification of angular cloud extents (FitzGerald 1968; Neckel & Klare 1980), the differential extinction values are typically larger than studies in which simpler angular bin sizes and shapes have been chosen (Arenou *et al.* 1992). This is because cells in which the angular size exceeds the cloud size are polluted with stars having little or no extinction, and these tend to make an observer underestimate the cloud's extinction value. It is found that the maximum differential extinction ranges from roughly 20 mag/kpc (for the high angular resolution study of Neckel & Klare) to 10 mag/kpc (for the intermediate angular resolution study of FitzGerald) to only 5 mag/kpc (for the low angular resolution study of Arenou *et al.*). The amount of visual extinction obtained

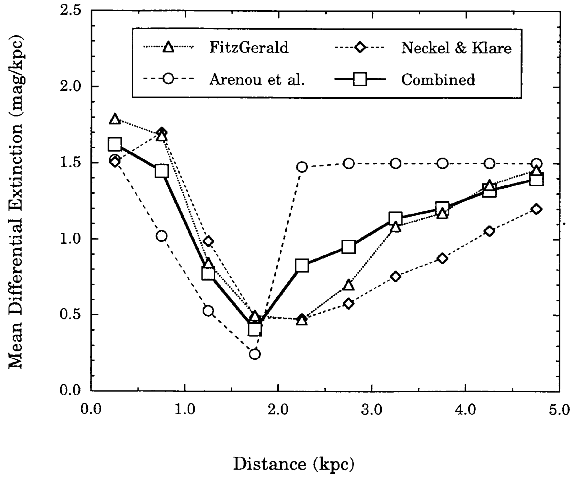

FIG. 10. Mean differential extinction a_v (mag/kpc) in the Galactic plane as a function of distance d(kpc), obtained from the studies of FitzGerald (1968), Neckel & Klare (1980), and Arenou, Grenon, & Gómez (1992). Studies with better angular resolution find larger average extinction values than those with poorer angular resolution. There is also a noticeable trend for average differential extinction to decrease with distance. Reasons for this are discussed in the text.

from each study correlates with that study's angular resolution.

An interesting systematic trend is also seen in Fig. 9; the mean differential extinction decreases with increasing distance from the Sun (up until the distance at which the statistical differential extinction value is introduced), such that the most opaque clouds are generally found nearby. It is more pronounced for low angular resolution studies than for high angular resolution ones.

Figure 10 more clearly demonstrates the trend, as the mean differential extinction is shown to decrease in all studies beyond distances of 1 kpc. It should be noted that the mean differential extinction returns to the (default) statistical value of 1.5 mag/kpc at larger distances (roughly 5 kpc) where the actual extinction has been poorly surveyed.

If we exclude the possibility that nearby clouds are really the most opaque ones, then a simple explanation for this trend is that the cloud filling factor decreases in cells of constant solid angle. This causes extinction from clouds to be averaged in with extinction from regions containing no clouds, decreasing the overall extinction in the cell. If we assume that a cell completely bounds a cloud at distance d_c having total extinction $A_v(d_c)$, that no other extinction is present, and that overall cell extinction is just an average of extinction measured at different locations within a cell, then the total extinction measured in the cell at distance d ($d > d_c$) is just $Av(d)=(d/d_c)^{-2}A_v(d_c)$. At large distances cells cover regions of space larger than typical cloud sizes so that extinction of these clouds is underestimated. The effect is more pronounced for cells so large that they never completely bound a cloud at any distance.

It is also possible that extinction variations across the angular surface of a cloud results in an underestimation of the cloud's extinction (since cloud structures are generally far from uniform). This is because the average cloud extinction will combine measurements made in high- and low-extinction regions. If we naïvely assume that a fraction of the cloud f has extinction A_v and that the remainder is transparent, then the measured extinction will be $A_{v\ \mathrm{meas}}=fA_v$ if the overall cell extinction is just an average of extinction measured at different cell locations. This effect will be small if the extinction variations are small across the cloud's surface, and will be balanced somewhat by cloud regions where the extinction is larger than average (provided this extinction is not too large to be measured). Variations in cloud extinction are thus smaller than those due to the cloud filling factor.

TABLE 1. Dilution of average differential extinction measurements due to inclusion of high-$|z|$ extinction measurements. At a particular distance, the differential extinction a_{cell} is obtained by averaging over a "typical" cell with a specific angular radius perpendicular to the Galactic plane, and normalized to the planar differential extinction value a_0. It is assumed that the extinction obeys a Parenago law [Eq. (1)] perpendicular to the Galactic plane with a scale height of 114 pc. This effect contributes somewhat to the high extinction observed nearby (see Fig. 10), but by itself does ot explain why the 1980 Neckel & Klare study exhibits the effect as well.

Study	Galactic plane cell radius	(a_{cell}/a_0) at 1 kpc	(a_{cell}/a_0) at 2 kpc
Neckel & Klare (1980)	1°	0.93	0.88
FitzGerald (1968)	6°	0.70	0.51
Arenou et al. (1992)	5°	0.74	0.56

It is possible that average extinction appears larger nearby because the angle subtended allows inclusion of measurements at larger $|z|$ distances (perpendicular to the Galactic plane), where extinction is lower. Assuming that studies have sampled a Parenago distribution centered on the Galactic plane, the average differential extinction a_{cell} measured at a distance d in a region with angular radius θ is given by

$$a_{\mathrm{cell}} \approx 2a_0x[I_1(x)-L_1(x)], \tag{6}$$

where $x=(d \tan \theta)/h$, h is the scale height of the extinction distribution, $I_1(x)$ is a first-order modified Bessel function, $L_1(x)$ is a first-order modified Struve function, and a_0 is the differential extinction in the Galactic plane. The average differential extinction values are shown in Table 1 for the three Galactic plane studies at distances of 1 and 2 kpc. When integrated over the line of sight, the size of this effect becomes smaller. The effect therefore contributes somewhat to the high extinction observed nearby, but is not as significant as the filling factor effect mentioned previously. At distances larger than 2 kpc, however, this effect contributes significantly to data collected in all of the studies.

Another bias can add to this effect: opaque clouds block more distant regions so that complete data are not available in these directions at large distances. This tends to bias the survey against more distant opaque clouds. Unfortunately, it is difficult to estimate the magnitude of this effect without a detailed analysis of instrumental biases in the surveys from which the extinction measurements have been made.

The lower differential extinction values obtained from low angular resolution studies, coupled with the observation that the overall extinction decreases with distance, indicate to us that the overall extinction is underestimated more by low angular resolution studies than by high angular resolution

TABLE 2. Average Galactic Plane differential extinction $\langle a_v \rangle$ (mag/kpc) as obtained from pertinent studies. Quoted mean errors do not include a systematic component that underestimates extinction at large distances (see text), but it is assumed that this error is negligible within the limiting distance of 1 kpc.

Study	$\langle a_v \rangle$ (mag/kpc)
Arenou et al. (1992)	1.1 ± 0.1
FitzGerald (1968)	1.7 ± 0.1
Neckel & Klare (1980)	1.7 ± 0.1
Average	1.5 ± 0.1

ones. One could argue that low angular resolution studies merely distribute extinction over larger cells, so that surrounding regions have correspondingly higher extinction. However, since all studies indicate that extinction decreases with distance, angular cell size clearly causes an underestimate in the extinction. Since angular cells are larger in low angular resolution studies, these studies should most severely underestimate the overall extinction.

In Table 2 we demonstrate how the computer code can be used to obtain a differential extinction in the Galactic plane of $a_v = 1.5$ mag/kpc. In obtaining this value, we have only sampled within 1 kpc of the Sun in order to avoid the aforementioned selection biases. Our results are in agreement with values obtained from specific studies (e.g., Guarinos 1991) and intermediate between the values of 1 mag/kpc (e.g., Carroll & Ostlie 1996) and 2 mag/kpc (e.g., Karttunen *et al.* 1994) often quoted in textbooks.

We suggest a method by which to correct the systematic extinction underestimation between 1 and 5 kpc. We estimate the unsampled planar differential extinction as function of distance (in 0.5 kpc bins) from Fig. 10; it is $\langle a_v(d) \rangle_{\rm unsampled} = 1.5 - \langle a_v(d) \rangle_{\rm sampled}$. A distance-dependent correction term $A_{vc}(d)$ is obtained by combining this differential extinction with Eq. (2). The corrected extinction $A_v(d) + A_{vc}(d)$ maintains a 1.5 mag/kpc average (by adding extinction in the undersampled regions proportional to that assumed missing), while still allowing for the identification of some opaque clouds beyond 1 kpc.

4.2 Extinction Perpendicular to the Galactic Plane

Figure 11 demonstrates the total visual extinction A_v (mags) in Galactic coordinates to a distance of 1 kpc as obtained from combining the results of all studies so that extinction from individual clouds identified in the high-latitude study is preferentially used when available. Despite the low angular resolution used in making this plot, a number of clouds and cloud complexes are seen.

It was stated earlier that the Parenago relation's smooth functional form does not accurately reproduce clumpy interstellar extinction variations. Figure 12 demonstrates the difference between our modeled extinction and that calculated from the Parenago relation [Eq. (1)], or

$$\Delta A_v = (A_v)_{\rm model} - (A_v)_{\rm Parenago}. \quad (7)$$

The values are calculated over the entire celestial sphere at a

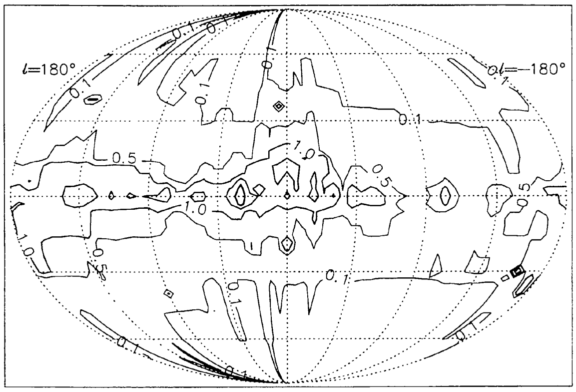

FIG. 11. Total visual extinction A_v (mag) in Galactic coordinates to a distance of 1 kpc as obtained from combining the results of all studies. When available, extinction from individual clouds identified in the high-latitude study is used. Otherwise, the results represent an average of available subroutines weighted equally. Contours in this plot are 0.1, 0.5, 1.0, 2.0, and 3.0 mag.

distance of 1 kpc, assuming a Parenago scale height of 114 pc. The extinction difference ΔA_v assumes positive values when our model extinction exceeds that of the Parenago relation, and takes on negative values when the extinction predicted by the Parenago relation exceeds that of our model. Figure 12 indicates that variations in ΔA_v are generally small over the celestial sphere, except in regions where well-defined clouds and cloud complexes are found.

The complexity of our model perpendicular to the Galactic plane is thus verified, so we now set out to establish its superiority to analytical models. As a simple check, we measure extinction in the directions of the Galactic poles. The total extinction toward the Galactic poles is summarized in Table 3. The results [A_v(NGP,SGP)$\approx 0.1 \pm 0.2$] are found to be in good agreement with values obtained elsewhere (see Burstein & Heiles 1982 for a summary) which indicate that A_v(NGP)$\approx$0.03 mag and A_v(SGP)$\approx$0.05 mag.

The extinction at high-Galactic latitudes can be computed

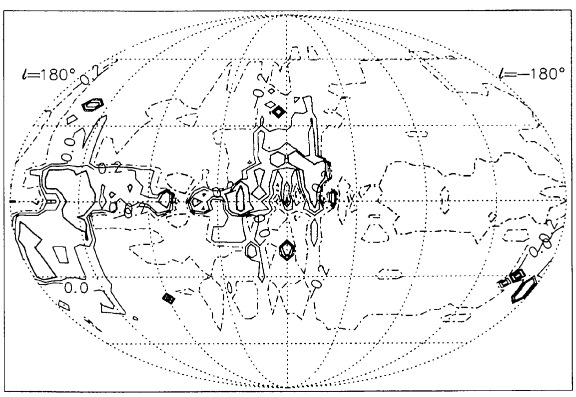

FIG. 12. Difference between the visual extinction A_v (mag) calculated by the computer model at 1 kpc (shown in Fig. 11) and the extinction predicted from the Parenago relation for an assumed scale height of 114 pc. Contours in this plot are −1.5, −0.5, −0.1, 0.0, 0.1, 0.5, and 1.5 mag, with negative values (less extinction than the Parenago Law predicts) plotted as increasingly thicker dotted lines and other values (more extinction than the Parenago Law predicts) plotted as increasingly thicker solid lines.

TABLE 3. Total extinction (mag) in the directions of the Galactic poles to a distance of 1 kpc from pertinent studies. Quoted errors are formal standard deviations, rather than the mean errors used in Table 1. These values compare favorably to those of other studies (summarized by Burstein & Heiles 1982), which indicate that A_v(NGP)≈0.03 mag and A_v(SGP)≈0.05 mag.

Study	Gal. North A_v (mag)	Gal. South A_v (mag)
Arenou et al. (1992)	0.1±0.2	0.1±0.2
Berdnikov & Pavlovskaya (1991)	0.3±0.4	0.3±0.4
FitzGerald (1968)	0.0±0.0	0.0±0.0
Penprase (1992)	0.0±0.2	0.0±0.2
Average	0.1±0.2	0.1±0.2

from the model and compared to extinction values obtained from 21 cm observations. Figure 13 compares results of the model to extinction calculated from the northern hemisphere portion of the Burstein & Heiles (1982) H I survey. Although the large formal errors of both studies indicate good agreement, we have subdivided the samples into high- and mid-Galactic latitude samples for further analysis.

Comparison of the high-latitude data suggests that our values are slightly higher than those obtained from 21 cm observations, although the most different values are those between $20° < b < 30°$. We suggest that the abrupt transition in the structure and size of clouds away from the Galactic plane might perhaps contribute to our extinction overestimation. Thus, smoothly-varying functional forms (such as those of FitzGerald and Berdnikov & Pavlovskya) would not recognize an abrupt drop in extinction if cloud sizes and structures abruptly changed, while studies with large cell sizes (such as Arenou *et al.*) would average lower extinction at high latitudes with higher extinction at lower latitudes.

We have chosen to not compare our model to *IRAS* observations. This is because (1) *IRAS* observations were already used to define boundaries for many high-latitude clouds (so a correlation must exist for these), and (2) survey incompleteness prevents us from obtaining extinction for previously-identified clouds (so it is clear that no correlation will exist for these).

4.3 *Extinction at Middle Galactic Latitudes*

Although the extinction model produces accurate results at high- and low-Galactic latitudes, the mid-latitude data ($7° \leq b \leq 16°$) are less accurate. This is to be expected, since the mid-latitude regions have not been surveyed as completely as other regions. This is due primarily to the complexity of Gould's Belt material, and to the transition in cloud structures from large complexes at low-Galactic latitudes to small clouds at high-Galactic latitudes. Although we have attempted to base our mid-latitude model on individual cloud measurements combined with a statistical description, we find that mid-latitude information is far from complete at this time.

The model incompleteness can be verified by examining mid-Galactic latitude measurements shown in Fig. 13. These indicate that our model slightly underestimates the extinction at these latitudes, in contrast to slightly overestimating the extinction at high latitudes (described previously).

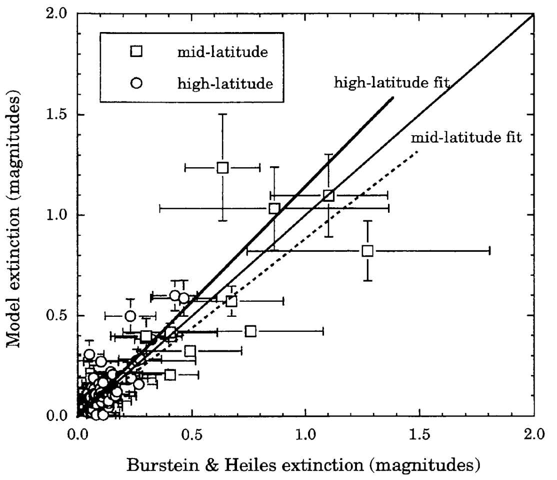

FIG. 13. Comparison of the extinction model at high-Galactic latitudes with extinction obtained from H I measurements by Burstein & Heiles (1982). Circles (fit by the dark solid line) indicate high-latitude measurements, while squares (fit by the dark dotted line) indicate mid-latitude measurements. There is good agreement between our model and the Burstein & Heiles extinction, although it appears that our model slightly overestimates extinction at high latitudes and underestimates it at mid latitudes.

5. CONCLUSIONS

We have produced a computational tool that can be used to identify large-scale visual Galactic extinction. The tool can be used (among other things) (1) as an aid in the statistical studies of Galactic stellar distributions (e.g., Myers & Hakkila 1996; Hallum *et al.* 1994), and (2) for correcting extragalactic measurements for interstellar extinction effects (e.g., van Paradijs *et al.* 1997). The tool has limitations as well; it is not designed to identify small scale (less than 1°) extinction variations. It is also a less outstanding (but still quite good) estimator at mid-Galactic latitudes ($7° \leq b \leq 16°$) and (without correction) at distances between 1 and 5 kpc in the Galactic plane; these are due to sampling incompletenesses.

We conclude that studies of large-scale interstellar extinction in the Galactic plane (whether analyzed here or not) generally tend to underestimate extinction at large distances (typically greater than 1 kpc). This is because (a) cells of predefined solid angle cause clouds to be progressively less conspicuous at larger distances, (b) the opacity of nearby clouds prevent more distant clouds from being accurately identified above the limiting magnitude of the survey, and (c) cells of predefined solid angle cause higher-Galactic latitude regions with lower extinction to be more heavily sampled at larger distances. Our subroutine provides the user with a statistical correction term that can correct for these systematic biases.

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